Detrended fluctuation analysis in multiparticle production

P. Mali[1],* S. Sarkar[1], S. Ghosh[1], A. Mukhopadhyay[1], and G. Singh[2]
[1] Department of Physics, University of North Bengal, Siliguri 734 013, India and
[2] Department of Computer and Information Science, SUNY at Fredonia, NY 14063, USA

The multifractal detreneded fluctuation analysis (MF-DFA) introduced in [1] is found to be a highly successful method in analyzing nonstationary stochastic processes. So far the method has been applied to different areas of statistical analysis, for instance see [2] and the references therein. In this paper we apply the technique to the pseudorapidity (η) distribution of shower tracks coming out of ^{28}Si-Ag(Br) events at an incident energy of 14.5A GeV. Each event has a shower track multiplicity $n > 50$. We compare the experiment with the prediction of the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) [3].

In the MF-DFA formalism first a "profile" function Y is to be determined out of the data points x_k as:

$$Y(i) = \sum_{k=1}^{i}[x_k - \langle x\rangle],\ i = 1, \ldots, N. \quad (1)$$

Then the profile $Y(i)$ is divided into $N_s \equiv int(N/s)$ segments of equal length s. Then the variance of each segment p with respect to the local trend:

$$F^2(p, s) = \frac{1}{s}\sum_{i=1}^{s}\{Y[(p-1)s+i]-y_p(i)\}^2, \quad (2)$$

is obtained. Here $y_p(i)$ represents the local trend for the segment p. We consider a linear trend of the event-wise local particle density i.e., $x_k = dn/d\eta$. The density distribution plot ($dn/d\eta$ against η) for a typical high multiplicity event is shown in Fig. 1. Finally, the qth order MF-DFA function is defined as:

$$F_q(s) = \left\{\frac{1}{N_s}\sum_{p=1}^{N_s}[F^2(p, s)]^{q/2}\right\}^{1/q} \quad (3)$$

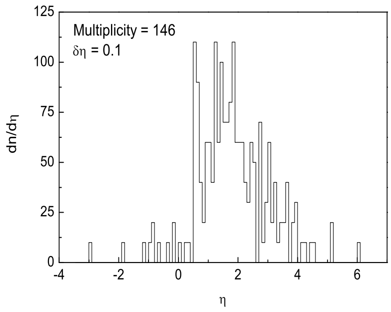

FIG. 1: A plot of particle density against η in a typical high multiplicity ^{28}Si-Ag(Br) event.

for any $q \neq 0$. For $q = 0$ the definition is modified as:

$$F_0(s) = \exp\left\{\frac{1}{2N_s}\sum_{p=1}^{N_s}\ln[F^2(p, s)]\right\}. \quad (4)$$

If the series x_k is a fractal one then $F_q(s)$ for large s and for all q would exhibit a power-law scaling behaviour like: $F_q(s) \sim s^{h(q)}$. In general, for a multifractal series the exponent $h(q)$ depends on q while for a monofractal series it is expected to be independent of q, i.e., $h(q) = H$, the Hurst exponent [4]. Moreover, for stationary series $h(2) = H$ [4]. Thus, one can distinguish the function $h(q)$ as the generalized Hurst exponent, which is related to the multifractal scaling exponent $\tau(q)$ as $\tau(q) = q\,h(q) - 1$. The multifractal singularity spectrum $f(\alpha)$ is determined via a Legendre transformation: $f(\alpha) = q\alpha - \tau(q)$, where $\alpha = \tau'(q)$.

Since the method, originally developed for a nonstationary time series of effectively infinite length, is applied to a series of finite length ($n \geq 50$), we average the MF-DFA fluctuation function $F_q(s)$ over the total number of events ($N_{ev} = 158$) in our sample. Fig. 2 shows the event averaged MF-DFA fluctuation functions

*Electronic address: provashmali@gmail.com

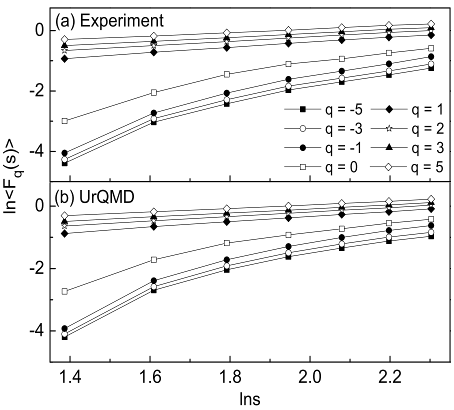

FIG. 2: Log-log plots of the event averaged MF-FDA fluctuation functions $F_q(s)$ with scale s

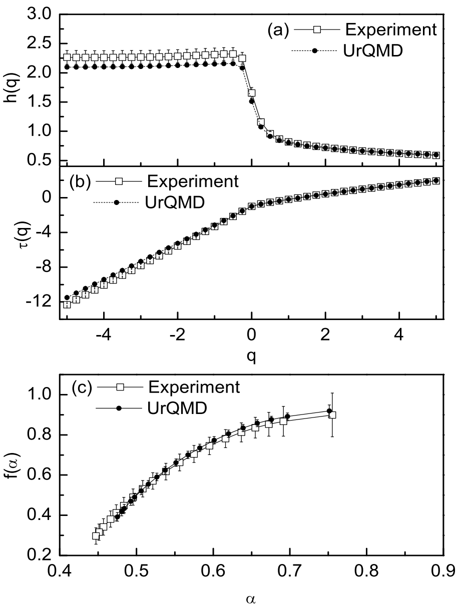

FIG. 3: (a) The generalized Hurst exponent $h(q)$, (b) the corresponding $\tau(q)$ exponents and (c) the multifractal singularity spectra.

$F_q(s)$ plotted against the scale s for several values of q. As expected both the experiment and the UrQMD generated plots follow the power-law type of scaling. The exponent $h(q)$ are calculated from the linear fits to the data points for $q = -5$ to $+5$. The order dependence of the generalized Hurst exponents is shown in Fig. 3(a) and the corresponding $\tau(q)$ exponent spectra and the multifractal spectra are shown, respectively, in Fig. 3(b) and 3(c). However, the complete $f(\alpha)$ spectrum could not be obtained, because of its unusual behaviour in the $q < 0$ region, as is also seen in a similar analysis [5]. The observed nonlinearity in the $h(q)$ and $\tau(q)$ spectra and the concave nature of the $f(\alpha)$-spectra are clear signatures of multifractality in the η-distribution of the event samples analyzed. The experiment and the UrQMD exhibit more or less similar trends but the degree of multifractality is a little weaker in the simulation. The results of this analysis are almost consistent with those of our previous multifractal analysis using a different technique [6].

In summary, we have applied the multifractal detreneded fluctuation analysis in order to characterize the η-distribution of charged particles emitted from ^{28}Si-Ag(Br) collisions at an incident energy of 14.5A GeV. The results of our analysis show multifractal nature in the η-distribution for both the experiment and the UrQMD simulation. From our preliminary results it is expected that the MF-DFA method will reliably characterize the multifractal pattern of the phase-space distribution in high-energy heavy-ion collisions.

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