

## Dynamical fluctuations in multiplicity distribution of particles produced in relativistic nuclear collisions

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### Introduction

Non-statistical fluctuations in high multiplicity events produced in relativistic particle-nucleus and nucleus-nucleus collisions have been extensively investigated by Takagi [1]. Miyamura and Tabuki [2] have also studied multiplicity distributions in different rapidity bins using scaled factorial moments,  $F_q$ , approach [3]. The bin size dependence of normalized factorial moments of the JACEE event [4] was analyzed by Bialas and Peschanski [5]. However, approaches, which are based on the concept of multi-fractals, appear to be the most promising as they are believed to be related to self-similar cascading, chaos, phase transitions, etc; [6]. Some interesting results on intermittency and multifractality have already been reported earlier [7]. But it always turned out that the experimental data do not show the expected linear behavior in  $\ln\langle n_q \rangle$  versus  $\ln\langle n \rangle$  plots. However, the trend may be explained successfully by Takagi's method.

In the present study, nature of dynamical fluctuations is investigated by analyzing experimental data on 200A GeV/c  $^{16}\text{O}$ -nucleus collisions.

### Mathematical Formalism

To investigate multifractality behaviour, we have adopted the formalism developed by Takagi. In this approach Takagi divided the full rapidity interval into 'm' bins of equal size,  $\delta\eta = \Delta\eta / m$ . In a single bin multiplicity distribution,  $P_n(\delta\eta)$ , for  $n=0,1,2,3 \dots$ , the inclusive rapidity distribution,  $dn/d\eta$ , is envisaged to be flat and  $P_n(\delta\eta)$  is visualized to be independent of the location of the bin. Let  $n_{ij}$  be the number of charged particles in  $j^{\text{th}}$  bin of  $i^{\text{th}}$  event and N the total number of events. The particle density,  $P_{ij}$ , is related to  $T_q(\delta\eta)$  in the following fashion:

$$T_q(\delta\eta) = \ln \sum_{j=1}^M \sum_{i=1}^m P_{ij}^q \quad \text{for } q > 0 \quad (1)$$

where q is order of the moment. This is similar to linear logarithmic function of resolution,  $R(\delta\eta)$ , having the form:

$$T_q(\delta\eta) = A_q + B_q \ln R(\delta\eta) \quad (2)$$

where  $A_q$  and  $B_q$  are independent of  $\delta\eta$  and for a given range of  $R(\delta\eta)$ . The generalized dimensions,  $D_q$ , are calculated from:

$$D_q = B_q / (q-1) \quad (3)$$

In order to measure the fluctuations, Takagi has modified Eq. (2) to have the following form:

$$\ln\langle n^q \rangle = A_q + (B_q + 1) \ln\langle n \rangle \quad (4)$$

where q is a positive real number.

The values of generalized dimensions,  $D_q$ , are obtained by obtaining the values of the slopes using Eq. (3) for  $q=2, 3, 4$  and  $5$ .

For investigating intermittency behavior we divide the rapidity into M smaller rapidity bins. The intermittency is given as:

$$F_q(\delta\eta) = \frac{1}{N_{evt}} \sum_{N_{evt}} \sum_{m=1}^M \frac{\langle n(n-1)\dots(n-q+1) \rangle}{\langle N \rangle^q} \quad (5)$$

M, determined by the above equation, is a positive integer. The intermittent behavior of the multiplicity distribution manifests itself as a power law dependence of factorial moment on the cell size for cell size  $\geq 0$ .

$$\langle F_q \rangle \propto M^{\alpha_q} \quad (6)$$

The exponent  $\alpha_q$  is the slope characterizing the linear rise of  $\ln\langle F_q \rangle$  with  $\ln M$  and is obtained as the slope. By obtaining the values of  $\alpha_q$ ,  $D_q$  values are determined using the relation:

$$D_q = \alpha_q / (q - 1) \quad (7)$$

$D_q$  and  $d_q$  are related as

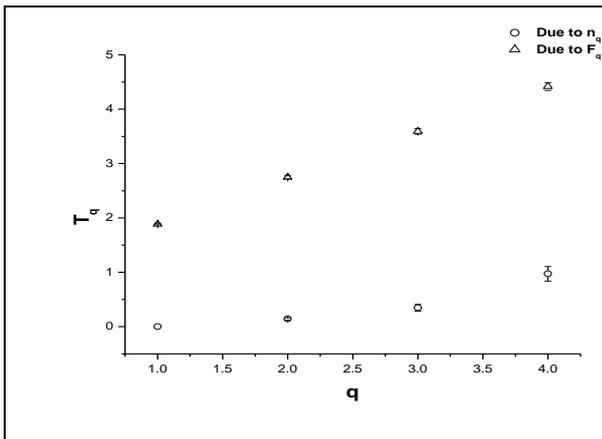
$$D_q = 1 - d_q \quad (8)$$

**Experimental details**

An emulsion stack, exposed to a beam of 200A GeV/c Oxygen nuclei from AGS, BNL is used in the present study. The events were scanned along the track method and space angles of the produced charged particle, were measured using the coordinate method. The other details of the experiment may be found in Reference [7]. A random sample comprising of 1165 interactions having  $n_h \geq 0$  produced in 200A GeV/c  $^{16}\text{O}$ -nucleus interaction are analyzed; where  $n_h$  represents the number of charged particles produced in an interaction with relative velocities,  $\beta(= v/c) < 0.7$ , where  $c$  represents speed of light in vacuum.

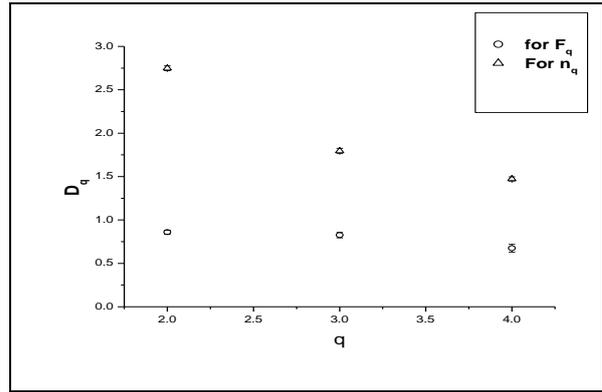
**Results and Discussions**

The values of  $T_q$  are plotted against  $q$  for different pseudorapidity intervals for our experimental data. It is seen from Fig.1 that variation of  $T_q$  with  $q$  is a linear one.



**Fig. 1** Variation of  $T_q$  with  $q$

The values of generalized dimensions,  $D_q$ , may be calculated from the slopes of the linear fits using Eqs. (4) and (7).



**Fig. 2** Variation of  $D_q$  with  $q$

The values of  $D_q$  are listed in the Table 1.

**Table: 1**

q	$D_q$	
	$F_q$	$n_q$
2	0.8590	$\pm 0.0222$
	2.7479	$\pm 0.0281$
3	0.8254	$\pm 0.0305$
	1.7955	$\pm 0.0296$
4	0.6752	$\pm 0.0444$
	1.4727	$\pm 0.0241$

**Conclusion:**

Strong dependence of  $T_q$  on the value of  $q$  for multifractal and intermittency approaches are observed. Finally, the values of  $D_q$  reveal the presence of multifractal geometry in the data.

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