Shear viscosity due to quark-pion interaction

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We have calculated the shear viscosity coefficient η of the strongly interacting matter in the relaxation time approximation, where a quasi particle description of quarks with its dynamical mass is considered from Nambu-Jona-Lasinio (NJL) model. In the relaxation time approximation, the η can be expressed as [1]

$$\eta = \frac{2\beta}{5\pi^2}\int_{M_k}^{\infty} d\omega_k \frac{(\omega_k^2 - M_k^2)^{5/2}}{\omega_k} \left[\frac{n_k^+(1-n_k^+)}{\Gamma} + \frac{n_k^-(1-n_k^-)}{\Gamma}\right] \quad (1)$$

where $n_k^{\pm} = \frac{1}{e^{\beta(\omega_k \mp \mu)}+1}$ are quark and anti-quark distribution functions. Following the quasi particle description of NJL model, the dynamical quark mass, M_k tends to be the current quark mass, m_k at very high temperature (or density) due to the gap equation,

$$M_k = m_k + 4N_f N_c G \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{M_k}{\omega_k}(1-n_k^+-n_k^-) \quad (2)$$

where $\omega_k = \sqrt{\vec{k}^2 + M_k^2}$. The collisional rate, Γ may be estimated from the one-loop self-energy graphs (shown in Fig. 1) of quark or anti-quark at finite temperature, which may be expressed (for $\mu = 0$) as

$$\Gamma = -\epsilon(k_0)\mathrm{Im}\Sigma^R(k = m_k, T) \ . \quad (3)$$

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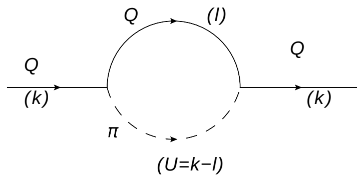

FIG. 1: The one-loop diagram of quark self-energy, where quark Q and pion π with four momentum l and $U = k - l$ respectively play the role of internal lines.

where

$$\mathrm{Im}\Sigma^R = \pi\int \frac{d^3\vec{l}}{(2\pi)^3}\frac{1}{4\omega_l\omega_U}L(l_0 = -\omega_l) (n_l^- + n_U)\delta(k_0 + \omega_l - \omega_U) \ . \quad (4)$$

with $n_l^-(\omega_l)$, $n_U(\omega_U)$ are Fermi-Dirac distribution function for quark and Bose-Einstein distribution function for π meson respectively.
The vertex factor L can be obtained from the quark-pion interaction Lagrangian,

$$\mathcal{L}_{\pi QQ} = \frac{-iM_k}{F_\pi}\overline{\psi}_f \vec{\pi}\cdot\vec{\tau}\gamma^5\psi_f \ . \quad (5)$$

where F_π is pion decay constant. The temperature dependence of η is shown by solid line in Fig. (2). In low temperature region, η is decreasing with increasing of T which is analogous to the behavior of liquid (From our daily life experience, we see that the cooking oil behaves like a less viscous medium when it is heated). Whereas in high temperature domain, η become an increasing function of T just like a system of gas. With the help of the two approximation in Eq. (1), we can understand mathematically these opposite nature of temperature dependence of η at two

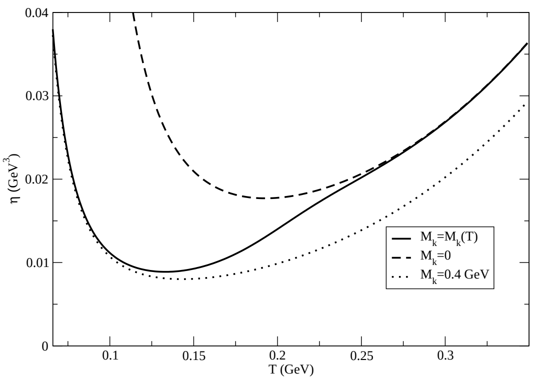

FIG. 2: The temperature dependence of shear viscosity (solid line) with respect to the outcome for the extreme limits of (i) $M_k = 0$ GeV (dashed line) and (ii) $M_k = 0.4$ GeV (dotted line).

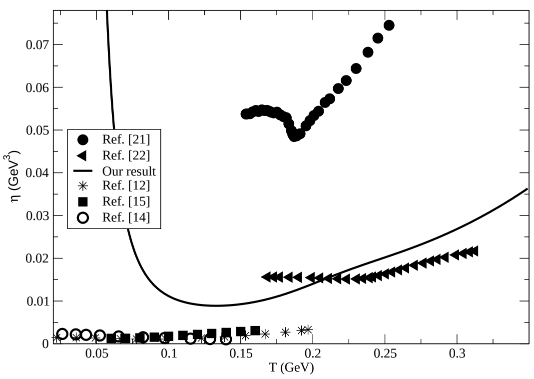

FIG. 3: Comparison of our results (solid line) with other results of Ref. [1](triangles) and Ref. [2](solid circles) [also the results of hadronic domain by Ref. [3](stars), Ref. [4] (open circles), Ref. [5] (squares)].

different temperature domain. At low temperature domain, we can write the Fermi-Dirac distribution functions (n_k^+) as a temperature independent step function. So only because of the term β outside the integration in Eq. (1), we will get $\eta \propto \frac{1}{T}$. Whereas in high temperature limit, $\eta \propto T^4$ because now our Fermi-Dirac distribution function can be replaced by Maxwell-Boltzmann distribution function and then we can do the integration analytically in terms of a gamma functions. However, the both approximations are considered for temperature independent Γ. The temperature from $T = 0.16$ GeV to 0.26 GeV in our η vs T graph is appeared to be most interesting region. To analyze this region we have run our calculation for two extreme limits of quark mass M_k in Eq. (1) - (i) $M_k = 0.4$ GeV (shown in dotted line) and (ii) $M_k = 0$ GeV (shown in dashed line). Now when we use the temperature dependence quark mass, $M_k = M(T)$ from the gap equation (2), then we see a smeared transformation of temperature dependency from the result of dotted curve to the result of dashed curve in the region of $T = 0.16 - 0.26$ GeV. This non-monotonic rising of η in this particular temperature region may be associated with quark-hadron phase transition, where the M_k approaches to m_k due to rapidly disappearing of quark condensate.

For comparison of the magnitude of our η with some previous calculations we have shown our results in Fig. (3) separately. The magnitude of η in our approach is very close to the results of Sasaki and Redlich [1] but underestimated with respect to the earlier estimation in NJL model by Zhuang et al. [2]. As we are focused on the η due to quark pion interaction therefore our approach may fail to estimate the η in low temperature domain, which is pointed by the standard calculations of η of hadronic matter [3–5].

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