Medium effects on the transport coefficients of a hot pion gas

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1. Introduction

The study of transport properties of strongly interacting matter has captured much attention in recent times. The interpretation of experimentally measured elliptic flow of hadrons in RHIC, by nearly ideal hydrodynamics with a small value of η/s, close to the quantum bound $1/4\pi$, provides much interest towards the transport coefficients, which characterize the response of an out of equilibrium system to certain disturbances.

Our aim is to determine the shear and bulk viscous coefficients and as well as the thermal conductivity for an interacting pion gas by solving relativistic transport equation in kinetic theory approach. For the dynamical input we need to construct a medium dependent $\pi\pi$ cross-section which gives noticeable effects on the temperature of viscous coefficients and thermal conductivity.

2. The transport coefficients in Chapman-Enskog approximation

The relativistic transport equation for the phase space distribution function $f(x,p)$ of a pion gas can be given by,

$$p^\mu \partial_\mu f(x,p) = C[f] \qquad (1)$$

The collision term $C[f]$, for a binary elastic collision is given by,

$$C[f] = \int d\Gamma_k \; d\Gamma_{p'} \; d\Gamma_{k'} [f(x,p')f(x,k') \times \{1+f(x,p)\}\{1+f(x,k)\} - f(x,p)f(x,k) \times \{1+f(x,p')\}\{1+f(x,k')\}] \; W, \qquad (2)$$

where the collision rate W is defined as $W = \frac{s}{2}\frac{d\sigma}{d\Omega}(2\pi)^6\delta^4(p+k-p'-k')$. The 1/2 factor denotes the indistinguishability of the initial state pions and the phase space factor is given by the notation $d\Gamma_q = \frac{d^3q}{(2\pi)^3 q_0}$.

After solving the relativistic transport equation in Chapmann-Enskog approximation we obtain the first approximation of shear viscosity, bulk viscosity and thermal conductivity as,

$$\eta = \frac{T}{10}\frac{\gamma_0^2}{c_{00}} \qquad (3)$$

$$\zeta = T\frac{\alpha_2^2}{a_{22}} \qquad (4)$$

$$\lambda = \frac{T}{3m_\pi}\frac{\beta_1^2}{b_{11}}, \qquad (5)$$

where the detailed expressions of the used quantities and the relevant calculations are discussed in detail in Ref [1].

3. The $\pi\pi$ cross-section with medium effects

To construct a $\pi\pi$ cross section we consider the scattering to proceed via ρ and σ meson exchange using the effective Lagrangian,

$$\mathcal{L} = g_\rho \vec{\rho}^\mu.\vec{\pi} + \frac{1}{2}g_\sigma m_\sigma \vec{\pi}.\vec{\pi}\sigma \qquad (6)$$

Introducing decay widths of ρ and σ mesons in the corresponding s-channel processes the matrix elements we obtain are,

$$M_{I=0} = 2g_\rho^2[\frac{s-u}{t-m_\rho^2} + \frac{s-t}{u-m_\rho^2}] + g_\sigma^2 m_\sigma^2[\frac{3}{s-m_\sigma^2+im_\sigma\Gamma_\sigma} + \frac{1}{t-m_\sigma^2} + \frac{1}{u-m_\sigma^2}]$$

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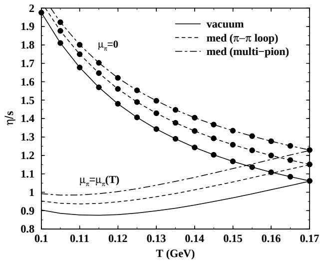

FIG. 1: The η/s as a function of temperature.

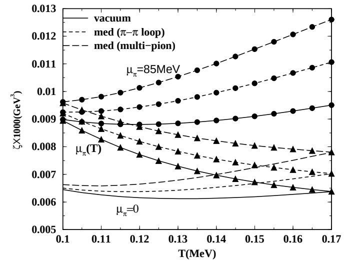

FIG. 2: ζ as a function of temperature.

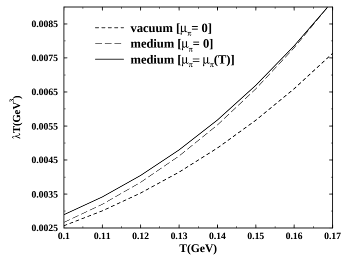

FIG. 3: λT as a function of temperature.

$$M_{I=1} = g_\rho^2[\frac{2(t-u)}{s-m_\rho^2+im_\rho\Gamma_\rho} + \frac{t-s}{u-m_\rho^2} - \frac{u-s}{t-m_\rho^2}] + g_\sigma^2 m_\sigma^2[\frac{1}{t-m_\sigma^2} - \frac{1}{u-m_\sigma^2}] \quad (7)$$

The differential cross-section is then obtained from $\frac{d\sigma}{d\Omega} = \overline{|M|^2}/64\pi^2 s$ where the isospin averaged amplitude is given by $\overline{|M|^2} = \frac{1}{9}\sum(2I+1)\overline{|I|^2}$.

In order to obtain the $\pi\pi$ cross section in the medium the ρ and the σ widths appearing in the matrix elements are replaced with the corresponding in-medium ones. The effect of the medium on ρ and σ propagation is quantified through its self-energy, which at finite temperature can be evaluated using a tool from thermal field theory called real time formalism. The detail of this calculation is given in Ref. [2]. The self-energy function Π which is related to the decay width by $k_0\Gamma(k) = -Im\Pi$, contains $\pi\pi$, $\pi\omega$, πh_1 and πa_1 loops. The mesons ω, h_1 and a_1 all having substantial 3π and $\rho\pi$ decay widths, can be considered as a multi-pion contribution to the ρ self-energy. The cross section obtained by using the in-medium ρ propagator suffers a small suppression of the peak for $\pi\pi$ loop and a larger effect for π-meson loop.

4. Results

We have shown the temperature dependence of the transport coefficients in our result section with different values of pion chemical potential. We have used a temperature dependent chemical potential [3], which increases with decreasing temperature using the fact of early chemical freeze out of pion gas in heavy ion collisions. Fig.1 shows the variation of η/s with T for $\mu_\pi = 0$ and $\mu_\pi = \mu_\pi(T)$, obtained from equation 3, where the effect of medium dependent cross section is prominently visible in the two set of curves. The variation of ζ with T is shown in the fig.2. The set of curves with $\mu_\pi = \mu_\pi(T)$ interpolates between the values of chemical freeze out (T=170 MeV, μ_π=0) and kinetic freeze out (T=100 MeV, μ_π=85 MeV). Finally we have plotted λT with temperature in fig.3, which also shows substantial medium effect on the thermal conductivity of pion gas.

References

[1] S. Mitra and S. Sarkar, Phys. Rev. D **87** (2013) 094026.

[2] S. Mitra, S.Ghosh and S. Sarkar, Phys. Rev. C **85** (2012) 064917.

[3] T. Hirano and K.Tsuda, Phys. Rev. C **66** (2002) 054905.