Bolzmann H-theorem and relativistic second-order dissipative hydrodynamics

Amaresh Jaiswal,* Rajeev S. Bhalerao, and Subrata Pal
Tata Institute of Fundamental Research, Mumbai - 400005, INDIA

Introduction

Application of the second-order viscous hydrodynamics to high-energy heavy-ion collisions has evoked widespread interest ever since a surprisingly small value for the shear viscosity to entropy density ratio η/s was estimated from the analysis of the elliptic flow data [1]. This led to the claim that the QGP formed at the Relativistic Heavy-Ion Collider (RHIC) was the most perfect fluid ever observed. A precise estimate of η/s is vital to the understanding of the properties of the QCD matter.

Priliminary lattice QCD studies for gluonic plasma suggests rather large values of bulk viscosity to entropy density ratio, ζ/s, near the QCD phase-transition temperature T_c [2] leading to large values of the bulk viscous pressure. This translates into negative longitudinal pressure which results in mechanical instabilities (cavitation) whereby the fluid breaks up into droplets [3]. Thus the theoretical uncertainties arising from the absence of reliable estimates for the second-order transport coefficients should be eliminated for a proper understanding of the system evolution.

We present here a formal derivation of the dissipative hydrodynamic equations where all the second-order transport coefficients get determined uniquely. This is achieved by invoking the second law of thermodynamics locally from the generalized entropy four-current given by Boltzmann H-function. Within one-dimensional scaling expansion, we demonstrate that the second-order transport coefficients derived here prevent the onset of cavitation even for rather large values of ζ/s estimated in lattice QCD.

*Electronic address: amaresh@tifr.res.in

Dissipative hydro equations

Hydrodynamic evolution of a medium is governed by the conservation equations for the energy-momentum tensor [4]

$$T^{\mu\nu} = \int dp\, p^\mu p^\nu f = \epsilon u^\mu u^\nu - (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{1}$$

where $dp = gd\mathbf{p}/[(2\pi)^3\sqrt{\mathbf{p}^2+m^2}]$, g and m being the degeneracy factor and particle rest mass, p^μ is the particle four-momentum and $f \equiv f(x,p)$ is the single particle phase-space distribution function. In the above tensor decomposition, ϵ, P are respectively energy density, pressure, and the dissipative quantities are the bulk viscous pressure (Π) and shear stress tensor ($\pi^{\mu\nu}$). Here $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projection operator on the three-space orthogonal to the hydrodynamic four-velocity u^μ defined in the Landau frame: $T^{\mu\nu}u_\nu = \epsilon u^\mu$. Energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, yields the fundamental evolution equations for ϵ and u^μ. The evolution equations for Π and $\pi^{\mu\nu}$ need to be specified to close the set of hydrodynamic equations.

Our starting point for the derivation of the dissipative evolution equations is the entropy four-current expression generalized from Boltzmann's H-function for particles obeying the Boltzmann statistics [4]

$$S^\mu(x) = -\int dp\, p^\mu f\,(\ln f - 1)\,, \tag{2}$$

For a system close to equilibrium, $f \equiv f_0(1+\phi)$, where $\phi \ll 1$. The equilibrium distribution function for vanishing chemical potential is defined as $f_0 = \exp(-\beta u\cdot p)$, where $u\cdot p \equiv u_\mu p^\mu$. The divergence of S^μ reads

$$\partial_\mu S^\mu = -\int dp\, p^\mu \big[\phi(1+\phi/2)(\partial_\mu f_0) + \phi(\partial_\mu\phi)f_0\big]. \tag{3}$$

To proceed further, ϕ needs to be specified. Grad's 14-moment approximation in orthogonal basis leads to [5]

$$\phi = \frac{\Pi}{P} + \frac{p^\mu p^\nu \pi_{\mu\nu}}{2(\epsilon + P)T^2} \ . \quad (4)$$

Performing the integrals in Eq. (3) as outlined in Ref. [5], we obtain

$$\partial_\mu S^\mu = \beta \pi^{\mu\nu} \left[\sigma_{\mu\nu} - \beta_2 \dot{\pi}_{\mu\nu} - \frac{4}{3}\beta_2 \theta \pi_{\mu\nu} \right] - \beta \Pi \left[\theta + \beta_0 \dot{\Pi} + \frac{4}{3}\beta_0 \theta \Pi \right], \quad (5)$$

where β_2, β_0 are thermodynamic functions.

The second law of thermodynamics, $\partial_\mu S^\mu \geq 0$, is guaranteed to be satisfied if

$$\pi^{\mu\nu} = 2\eta \left[\sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\langle\mu\nu\rangle} - \frac{4}{3}\beta_2 \theta \pi^{\mu\nu} \right],$$
$$\Pi = -\zeta \left[\theta + \beta_0 \dot{\Pi} + \frac{4}{3}\beta_0 \theta \Pi \right]. \quad (6)$$

where the coefficients of shear and bulk viscosity satisfy $\eta, \zeta \geq 0$. The shear and bulk relaxation times defined as

$$\tau_\pi = 2\eta\beta_2, \quad \tau_\Pi = \zeta\beta_0, \quad (7)$$

can be obtained directly from the transport coefficients β_2 and β_0 which are determined explicitly in the above derivation:

$$\beta_2 = \frac{3}{(\epsilon + P)} + \frac{m^2\beta^2 P}{2(\epsilon + P)^2}, \quad \beta_0 = \frac{1}{P}. \quad (8)$$

Results and discussions

The coupled differential equations, $\partial_\mu T^{\mu\nu} = 0$ and (6), are solved in the 1-D Bjorken model for $T_0 = 310$ MeV, $\tau_0 = 0.5$ fm/c, $\eta/s = 1/4\pi$ and lattice QCD equation of state [6]. For ζ/s, at $T > T_c$ we have used the lattice QCD results of Meyer [2] and for $T < T_c$, the results of the hadron resonance gas model.

In the absence of any reliable prediction for the bulk relaxation time τ_Π, it has been customary to keep it fixed or set it equal to the shear relaxation time τ_π [3]. Figure 1(a) compares the Π evolution for the three cases: (i) τ_Π obtained here in Eq. (7), (ii) $\tau_\Pi = 1$ fm/c, and (iii) $\tau_\Pi = \tau_\pi$. For cases (ii) and (iii), the rapid increase in ζ/s at T_c causes $|\Pi|$ to increase leading to vanishing longitudinal pressure ($P_L = (P + \Pi - \pi)$) and cavitation [3]. In contrast, with our τ_Π, i.e. case (i), this rise in ζ/s is compensated by an increase in τ_Π thereby slowing down the evolution of Π. This behavior prevents the onset of cavitation, which is also evident in Fig. 1(b), and guarantees the applicability of hydrodynamics with bulk and shear viscosity up to temperatures well below T_c into the hadronic phase.

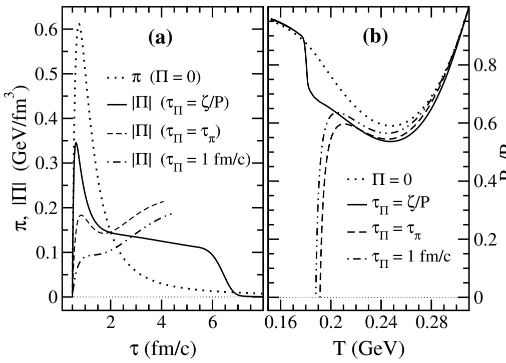

FIG. 1: (a) Time evolution of shear stress in the absence of bulk ($\Pi = 0$) and magnitude of bulk stress for $\tau_\Pi = \zeta/P$, $\tau_\Pi = \tau_\pi$ and for a constant $\tau_\Pi = 1$ fm/c. (b) Temperature dependence of pressure anisotropy, $P_L/P_T \equiv (P + \Pi - \pi)/(P + \Pi + \pi/2)$, for these four cases. The results are for initial $T = 310$ MeV, $\tau_0 = 0.5$ fm/c and $\eta/s = 1/4\pi$. The evolution is stopped when P_L vanishes.

References

[1] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007).

[2] H. B. Meyer, Phys. Rev. Lett. **100**, 162001 (2008).

[3] K. Rajagopal and N. Tripuraneni, JHEP **1003**, 018 (2010).

[4] S.R. de Groot, W.A. van Leeuwen, and Ch.G. van Weert, *Relativistic Kinetic Theory — Principles and Applications.*

[5] A. Jaiswal, R. S. Bhalerao and S. Pal, Phys. Rev. C **87**, 021901(R) (2013)

[6] A. Bazavov *et al.*, Phys. Rev. D **80**, 014504 (2009).