Magnetic interaction and thermal conductivity in degenerate QED plasma

S. Sarkar[1*] and A. K. Dutt-Mazumder[2]
[1 2] High Energy Nuclear and Particle Physics Division,
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700 064, INDIA

Introduction

Study of different transport and relaxation properties of QCD plasmas are of interest in different contexts specially in astrophysical situations such as neutron stars, white dwarfs etc.. Determination of thermal conductivity of degenerate electron matter has been a subject of serious investigation for the last several decades. Cooling mechanism of a new born star is governed mainly by two mechanisms. After neutrino emission core of the star cools quickly, but the crust remains hot. Hence, a temperature gradient is set up between the crust and the core. Thermal energy flows from the crust to the core resulting to thermalization. In the present work we intend to calculate the thermalization time scale which intimately related to the thermal conductivity coefficient. We also reveal the role of magnetic interaction in thermal conductivity.

Formalism

Present formalism is relevant in the context of neutron star crust which mainly contains degenerate electrons (e) and ions (i). Degenerate electrons constitute an ideal Fermi gas. Electron thermal conductivity (κ_e) is related to κ_{ee} and κ_{ei} *via* the following expression,

$$\frac{1}{\kappa_e} = \frac{1}{\kappa_{ei}} + \frac{1}{\kappa_{ee}}, \qquad (1)$$

where $\kappa_{ei,ee} \propto \frac{1}{T\nu_{ei,ee}}$. $\nu_{ei,ee}$ are the e-e and e-i collision frequencies. To calculate κ_e of the ideal Fermi gas we appeal to the Boltzmann equation. In absence of external force and presence of weak temperature gradient the equation takes the following form $\mathbf{v_p}.\nabla_\mathbf{r} f_p = -\mathcal{C}[f_p]$. $\mathcal{C}[f_p]$ describes the collision integral. Due to the presence of weak temperature gradient Fermi-Dirac distribution functions deviate from equilibrium distribution functions f_i which can be written as $\widetilde{f_i} = f_i + \frac{\partial f_i}{\partial \epsilon_i}\Phi_i \frac{\nabla T}{T}$. The second term with Φ measures the deviation from equilibrium. According to the variational principle the thermal conductivity is given by the maximum of the following equation

$$\begin{aligned} \frac{1}{\kappa_{ee}} \geq & \left(\nu \int_p \frac{(\epsilon_p - \mu)}{T} v_z f_p (1-f_p)\Psi_p\right)^{-2} \\ \times & \ \nu\nu' \int_{p,p',k,k'} f_p f_k (1-f_p')(1-f_k') \\ \times & \ (2\pi)^4 \delta^4(p+k-p'-k')|M|^2 \\ \times & \ \frac{(\Psi_p + \Psi_k - \Psi_{p'} - \Psi_{k'})^2}{4}, \end{aligned} \qquad (2)$$

Φ is given by the minimum of the above equation and the minimal value is Ψ. For the present purpose here we consider the simplest trial function $\Psi_p \propto (\epsilon_p - \mu)v_z$[1].

To proceed further one needs to know the interaction. Here, we consider only the electron-electron scattering,

$$|M|^2 = 32e^4 \left[\frac{1}{(q^2+\Pi_L)} + \frac{(1-x^2)\cos\phi}{(q^2-\omega^2+\Pi_T)}\right]^2 . (3)$$

In the above equation the medium modified photon propagator contains the polarization functions $\Pi_L(q,\omega)$ and $\Pi_T(q,\omega)$, which describe plasma screening of interparticle interaction by longitudinal and transverse plasma perturbations, respectively.

To perform the momentum integration in the phase-space factor in Eq.(2) one needs to know the dispersion relation. In presence

*Electronic address: sreemoyee.sarkar@saha.ac.in

of the medium the fermionic dispersion relation gets modified due to the inclusion of the fermion self-energy $\omega = E_p(\omega) - \mathrm{Re}\Sigma(\omega, p(\omega))$. For this one needs to know the fermion self-energy. The fermion self-energy for the QCD matter has already been quoted in [2]. In case of electrons at low temperature with next to leading order (NLO) correction it is given by the following [1],

$$\frac{d\mathrm{k}}{d\epsilon_k} = 1 + \frac{e^2}{12\pi^2}\log\left(\frac{4}{\pi\lambda}\right) + \frac{2^{2/3}e^2\lambda^{2/3}}{9\sqrt{3}\pi^{7/3}} \cdot (4)$$

The final expression for the electron thermal conductivity now takes the following form [1],

$$\kappa_{ee} = \left[\frac{\mathcal{C}}{T^2}(1+\beta)\left\{2\lambda^2\zeta(3) + \frac{(2\pi)^{2/3}}{3}\lambda^{8/3}\zeta\left(\frac{11}{3}\right)\Gamma\left(\frac{14}{3}\right) + \frac{\pi^5}{15}\lambda^3\right\}\right]^{-1}, \quad (5)$$

where, $\mathcal{C} = \frac{3e^4}{4\pi^5}$. Unlike Fermi-liquid result where, $\kappa_{ee} \propto 1/T$, here the temperature dependence is non-analytical, anomalous in nature [1]. κ_{ee} involves fractional powers in (T/m_D) coming from the medium modified phase space factor as shown above.

For the estimation of relaxation time defined as $\tau_\kappa = \frac{3\kappa}{C_v}$, the other quantity which we require is the specific heat. For the degenerate electron gas the form of specific heat is given in [1, 2].

With Eq.(5) and specific heat quoted in Ref.[2] the relaxation time for thermal conduction is expressed as follows [1, 2],

$$\tau_{\kappa_{ee}} = 3\kappa_{ee}/\left[\frac{\mu^2 T}{3} + \frac{m_D^2 T}{36}\left(\ln\left(\frac{4}{\pi\lambda}\right) + \gamma_E - \frac{6}{\pi^2}\zeta'(2) - 3\right) - 40\frac{2^{2/3}\Gamma\left(\frac{8}{3}\right)\zeta\left(\frac{8}{3}\right)m_D^3}{27\sqrt{3}\pi^{7/3}}\lambda^{5/3} + 560\frac{2^{1/3}\Gamma\left(\frac{10}{3}\right)\zeta\left(\frac{10}{3}\right)m_D^3}{81\sqrt{3}\pi^{11/3}}\lambda^{7/3}\right]. \quad (6)$$

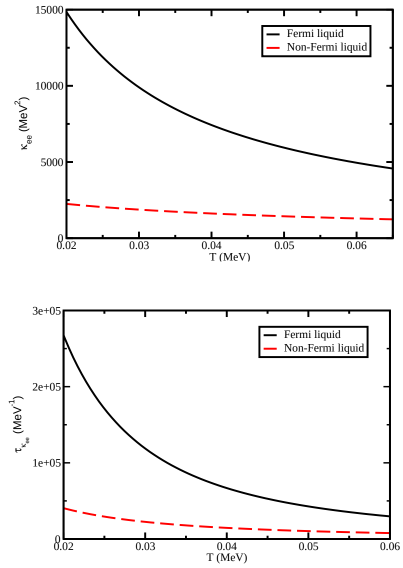

FIG. 1: Temperature dependence of κ_{ee} and $\tau_{\kappa_{ee}}$.

The thermal relaxation time upto NLO terms contains some anomalous fractional powers originated from the transverse interaction. This in turn changes the temperature dependence of $\tau_{\kappa_{ee}}$ non-trivially from the Fermi liquid result ($\tau_{\kappa_{ee}} \propto 1/T^2$).

In Fig.(1) we have plotted κ_{ee} and $\tau_{\kappa_{ee}}$ with temperature using Eqs.(5) and (6). From the plots it can be seen that inclusion of both the medium modified propagator and β decrease the value of both κ_{ee} and $\tau_{\kappa_{ee}}$. It shows strong deviation from the Fermi liquid results. This has serious implication on the total electron conductivity κ_e. Magnetic interaction decreases κ_{ee} which in turn increases the electron-electron collision frequency as can be seen from Eq.(1). Thus to the total electron thermal conductivity electron-electron scattering dominates over electron-ion scattering.

References

[1] S. Sarkar and A. K. Dutt-Mazumder, Phys. Rev. D **87**, 076003, (2013).

[2] A. Gerhold, A. Ipp and A. Rebhan, Phys.Rev.D **70**, 105015 (2004).