Gluon plasma thermodynamics: from transition region to SB limit

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Introduction

There are various phenomenological models to reproduce various equation of states of non-ideal quark gluon plasma (QGP) obtained in lattice QCD. Quasiparticle model (qQGP) is such a phenomenological model that explain the non-ideal behaviour seen in lattice simulation of QCD and relativistic heavy ion collisions. Different varieties of quasiparticle models are there, depending on different forms of thermal masses and running coupling constants. Here we discuss statistics and thermodynamics of pure SU(3) gluon plasma using two widely studied thermodynamically consistent quasiparticle models and try to fit the lattice results of precision SU(3) thermodynamics for a large temperature range (up to $1000T_c$)[1]. First quasiparticle model is the thermodynamically consistent quasiparticle model with reformulated statistical mechanics (QPM I)[2,3]. Second quasiparticle model is also thermodynamically consistent but using standard statistical mechanics without any reformulation (QPM II)[3]. We also try to predict the behavior of the quantities like, specific heat capacity $C_V$ which is the only thermodynamic fluctuation measure in pure gluon gas and the kinetic variable asso- ciated with it, the speed of sound $c_s$ using these models upto $1000T_c$. Hence we have a complete phenomenological description of the Equation of state of QGP from the phase transition through the perturbative region up to Stefan-Boltzmann limit.

Quasiparticle models

In the quasiparticle model our assumption is that, in the QGP, at finite temperature, instead of real quarks and gluons with QCD interaction, one can consider the system to be made up of non interacting quasiparticles, i.e., quasi-quarks and quasi-gluons, with temperature dependent effective mass. The simple dispersion relation for energy $\epsilon_k$ and momentum $k$ of the quasiparticle is $\epsilon_k = \sqrt{k^2 + m^2(T)}$. $m(T)$ is the temperature dependent mass, effective mass or thermal mass. The effective gluon mass taken in QPM I and QPM II are approximate two different limits of an exact dispersion relation given in [5] and is taken as $m^2(T) = \frac{g_1^2(T)T^2}{2}$ and $m^2(T) = \frac{g_2^2(T)T^2}{3}$ respectively. $g_1^2(T)$ is the one-loop order running coupling constant given by,

$$ g_1^2(T) = \frac{48\pi^2}{11N_c \ln(\frac{T_c}{T}) + \frac{T}{T_c})^2} $$

with $T_s/T_c$ as the phenomenological regularization and $T_c/\lambda$ represents the usual regularization scale parameter $\Lambda$. In QPM I, incorporating the additional medium contribution, the pressure $p$ and energy density $\varepsilon$ for a system of quasiparticles can be written in a thermodynamically consistent manner as follows

$$ p(T) = \frac{df}{6\pi^2} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m^2(T)}} - B(T), \quad (2) $$

$$ \varepsilon(T) = \frac{df}{2\pi^2} \int_0^\infty dk \frac{k^2\sqrt{k^2 + m^2(T)}}{e^{\frac{1}{2}\sqrt{k^2 + m^2(T)}} - 1} + B(T) \quad (3) $$

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FIG. 1: $P/T^4$ and $\varepsilon/T^4$ as a function of $T/T_c$ for gluon plasma from QPM I, QPM II are plotted and compared with lattice data. $C_s^2$ and $C_V/T^3$ as a function of $T/T_c$ are also plotted using the models.

The second term in eqn (2) and (3) represent the medium contribution i.e., $B(T) = \lim_{V \to \infty} E_0 V$, where $E_0$ is the vacuum energy in the absence of quasiparticle excitations or zero point energy. These equations involves a temperature dependent extra function $B(T)$ inorder to satisfy the relation $s = \partial p/\partial T$. It is used to maintain the thermodynamic consistency, i.e., to satisfy the relation $\varepsilon + p = T s$. In ref.[4], the author pointed out the reason for the thermodynamic inconsistency of the quasiparticle description of Peshier et al. If the particle mass is temperature dependent, the relation between the pressure and grand partition function may not be true. So here in QPM II we start from the definition of average energy in the grand canonical ensemble,

$$\varepsilon = \frac{1}{V} \sum_k \epsilon_k e^{-\frac{\epsilon_k}{T}}.$$  \hspace{1cm} (4)

The pressure can be derived from the thermodynamic relation $\varepsilon(T) = T \frac{\partial P}{\partial T} - P$.

**Results and Discussion**

From the previous papers [2-4] we find that these models successfully reproduce the lattice data up to $5T_c$. Now it is interesting to see that the models, QPM I and QPM II fits the recent lattice data [1] reasonably well in the large temperature range also, i.e., from 1 to 1000$T_c$. In fig. 1 we can see that, the pressure and energy density from the two models fits the lattice result reasonably well. Once we know the energy density and the pressure one can easily evaluate the speed of sound squared, defined as $C_s^2 = \frac{\partial p}{\partial \varepsilon}$ and the specific heat at constant volume $C_V = \frac{\partial \varepsilon}{\partial T}$. Those are also plotted in fig.1 from $T_c$ to 1000$T_c$. It is found that $C_s^2$ has sharp rise near $T_c$, flattened and reaches a value close to the SB limit of 1/3. $C_V/T^3$ has sharp peak at $T_c$ and approaches the corresponding SB limit at high temperature but shows significant deviation from it. It is found that QPM I and QPM II are good in explaining the lattice result from 1 to 1000$T_c$. So our study give extreme confidence in quasiparticle models being used as phenomenological models for the study of QGP in large temperature range, i.e., from $T_c$, through the perturbative region up to Stefan-Boltzmann limit.

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**References**