Thermodynamics of non-ideal QGP using Mayers cluster expansion method

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Introduction

The Quark gluon plasma (QGP) is the state in which the individual hadrons dissolve into a system of free (or almost free) quarks and gluons in strongly compressed system at high temperature. A systematic method of expansions, in the case of real gases obeying classical statistics, was developed by Mayer and his col laborators and known as the method of cluster expansions[1]. Mayers classical cluster expansion method was used for a QCD potential without screening [2,3]. B. Sheikholeslami-Sabzevari uses a different approach to find the equation of state which relates the particle density of heavy quarks to the temperature by considering homogeneous QGP [4]. The method gives a criterion for the calculation of temperature, where heavy quarkonium suppression and quarkonium enhancement occurs. This method is applicable only to a plasma of heavy quarks, i.e., charm and bottom, when considering the phase transition to $J/\psi(c,\bar{c})$ and $\Upsilon(b,\bar{b})$ mesons, respectively.

The present paper aims to calculate the critical temperature at which a non-ideal three quark plasma condenses into droplet of three quarks (ie, into a fluid of baryons e.g: $\Omega^-(sss)$) using Mayers cluster expansion method. Inorder to find out thermodynamic properties of three particle system we apply cornell potential into the bipolar coordinate method [5]. We intend to obtain a more accurate equation of state for quark plasma by considering the second and third virial coefficient. The obtained temperature of phase transition is then compared to the critical temperature calculated by conventional methods, like finite temperature QCD phase transitions. Here plasma is assumed to be homogeneous and containing classical gas of individual particles with a Coulomb plus a confining potential. The quarks (charm and bottom) are considered to be heavy enough for applying the non-relativistic approximation.

Three particle droplet formation by clustering of particles

In terms of mayer f-function, we form a cluster which is a collection of points connected directly or indirectly[1].

$$f_{ij} = 1 - e^{-\beta U(r_{ij})} \quad (1)$$

Here $U(r_{ij})$ is the Cornell potential between quarks, can be expressed as

$$U(r,T) = -\sigma r_D(1 - e^{-\frac{r}{r_D}}) + \frac{\alpha}{r} e^{-\frac{r}{r_D}} \quad (2)$$

Where the coupling constant α, string tension σ and the screening length r_D are temperature dependent.

When consider the clustering of quark-antiquark pairs to mesons we take second cluster integral [4].

$$b_2 = 2\pi \int_{r=0}^{\infty} [1 - e^{[-\beta(\sigma r_D + \frac{\alpha}{r})e^{-\frac{r}{r_D}}]}] r^2 dr \quad (3)$$

The second virial coefficient is written as

$$a_2 = -b_2 \quad (4)$$

For the cluster of three flavours, may be formed in any of the four ways. By the irreducible 3-cluster we mean the 3-particle graph which is multiply-connected. The third virial coefficient is

$$a_3 = 4b_2^2 - 2b_3 \quad (5)$$

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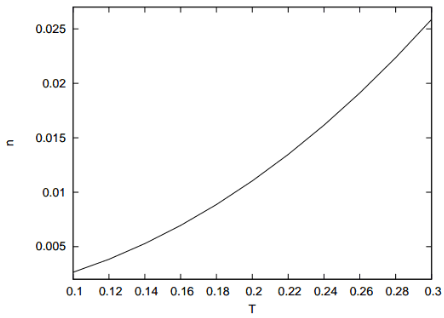

FIG. 1: Plot of n (particle number density) as a function of T(temperature) using the Cornell potential for two-flavor system.

In order to evaluate b_3 we introduce bipolar coordinates, by fixing the positions of the particle 1 and 2 and particle 3 takes all possible position and also using the technique of Jacobian transformation [5].

$$a_3(T) = \frac{8\pi^2}{3} \int_{r=0}^{\infty} r_{12}^2 f(r_{12}) \int_{r=0}^{\infty} r_{13}^2 f(r_{13}) \int_{r=-1}^{1} f(\sqrt{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}\mu}) d\mu dr_{12} dr_{13} \quad (6)$$

where $\mu = \cos(\phi)$.

The special potential given by Cornell potential (2) has used to calculate $a_2(T)$, $a_3(T)$ using Mathematica program.

Critical temperature for l-particle droplet formation (i.e., into a fluid of mesons and baryons) can be calculate using the relation [4]

$$T_{C_l} = \frac{2\pi}{M_l} \left[\frac{n}{l b_l} \right]^{\frac{2}{3l}} \quad (7)$$

where M_l is the mass of the particles of this species($l = 2$ for mesons and $l = 3$ for baryon).

Result and Discussion

Seventy years of experimental effort has revealed that quarks tend to cluster in quark-antiquark pairs called mesons, triplets of quarks called baryons. By considering this classical method at high temperature there can exist plasma of heavy quark and antiquark and it can condense into mesons and baryons at a critical temperature. To calculate QGP phase transition by incorporate lighter quarks we should extend our work into relativistic and quantum regime. There is also a possibility to incorporating fourth cluster integral in cluster of four particle droplet (four-quark matter). The recent viewpoint of two separate groups, both reporting in Physical Review Letters, have seen evidence for a strange particle, called $Z_C(3900)$ [6]. Z_C is a four-quark matter.

The main advantage of above mentioned method is that we can apply the classical particle picture to the quarks and investigate phase transitions in a QGP. In Fig. 1 we plot n(particle number density) versus T for two-flavor system. The equation of state found here shows the occurence of heavy quarkonium at $T_c = 150 - 200 MeV$. The equation also indicates that at future colliders with $T_c \geq 260 MeV$ the situation will not change.

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