Strong CP Problem and Axion Phenomenology

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Introduction

The theory of strong interaction, i.e., Quantum Chromo Dynamics (QCD), at the time of its inception, was associated with U(1) axial or $U(1)_A$ problem related with the observed hadronic spectrum. The situation persisted, till, during the seventies, 't Hooft [1] established that a θ term $\mathcal{L}_\theta \propto \frac{1}{M'} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ (where M' is the symmetry breaking scale and $F^{(a)\mu\nu}$'s are the field strength tensors for the gauge fields) should be added to the QCD Lagrangian because of instanton effects. Presence of the same however violates **CP** in the QCD sector and predicts neutron dipole-moment, that is experimentally observed to be $|d_n| < 3 \times 10^{-26}$ ecm; requiring θ to be fine tuned to $\leq 10^{-10}$. This fine tuning came to be known as the strong **CP** problem. To ameliorate this problem a spontaneously broken (with symmetry breaking scale M'=4M) global $U(1)_{PQ}$ symmetry, was introduced by R. Peccei and H. Quinn [2],S. Weinberg [3] and F. Wilczek [4], called Peccei-Queen symmetry, with axion (a(x)) being the pseudo-Nambu-Goldstone Boson of $U(1)_{PQ}$. Axions further couple to photons via anomalous fermionic triangle diagrams. Through nonperturbative effects it further generates a mass m_a. The resulting axion photon Lagrangian turns out to be:

$$\mathcal{L} = -\frac{1}{4}\mathrm{F}^{\mu\nu}\mathrm{F}_{\mu\nu} + \frac{1}{4M}\mathrm{F}^{\mu\nu}\tilde{\mathrm{F}}_{\mu\nu} + \frac{1}{2}\left(\partial_\mu a \partial^\mu a - m_a^2 a^2\right), \qquad (1)$$

Since the time of it's inception, there has been efforts to find out the symmetry breaking scale of $U(1)_{PQ}$ and the mass m_a of axion, because it not only provides a scheme to solve strong CP problem, but also can be a candidate to solve the cold dark matter problem in cosmology. *Being motivated by this, in this note we try to look for possible optical signatures of axion, in the limit of vanishing axion mass.* The Axion photon coupling term, allows it to oscillate into a photon and back in an external magnetic field $\mathcal{B}$. Therefore a magnetized vacuum would behave like a dichoric one if axions exist in nature. To study this effect we derived the equation of motion for axion a(x), the gauge fields, polarized perpendicular $A_\perp$ and parallel $A_{||}$ to the external magnetic field, in Lorentz gauge. The resulting equations, in matrix form, are:

$$\begin{pmatrix} (\omega^2+\partial_z^2) & 0 & 0 \\ 0 & (\omega^2+\partial_z^2) & ig\mathcal{B}\omega \\ 0 & -ig\mathcal{B}\omega & (\omega^2+\partial_z^2) \end{pmatrix} \begin{pmatrix} A_\perp(z) \\ A_{||}(z) \\ a(z) \end{pmatrix} = 0. \quad (2)$$

The solutions to the above equations, can be found out by solving equation [2] in diagonalized form and then inverting back. The solutions of the equations of motions, assuming a(0)=0, can further be written down, by defining $\Omega_- = (\sqrt{\omega^2 + g\mathcal{B}\omega} - \sqrt{\omega^2 - g\mathcal{B}\omega})$, $\Omega_+ = (\sqrt{\omega^2 + g\mathcal{B}\omega} - \sqrt{\omega^2 - g\mathcal{B}\omega})$ and $\mathcal{A}(z) = A_{||}(0)\left[2 + 2cos\left[\Omega_- z\right]\right]^{\frac{1}{2}}$ and assuming $\omega \sim k$ as:

$$\begin{aligned} a(t,z) &= \mathcal{A}(z)e^{-i(\omega t - \Omega_- z)}, \\ A_{||}(t,z) &= \mathcal{A}(z)e^{-i(\omega t - \Omega_+ z)}, \text{ and} \\ A_\perp(t,z) &= A_\perp(0)e^{i(\omega t - kz)}. \end{aligned} \qquad (3)$$

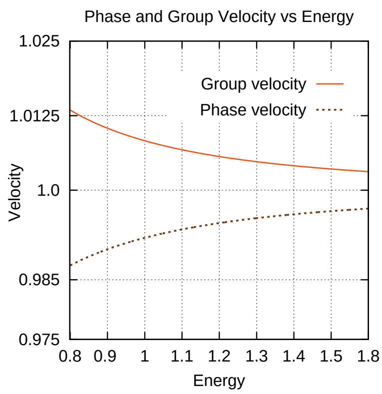

FIG. 1: The plot of Group Velocity v_g, and Phase Velocity v_p, versus energy. Group and phase velocities are plotted in natural units along y axis. And energy is plotted along x axis in units of 10^{-15}GeV.

In order to perform polarimetric analysis, one can find out the stokes parameters, defined in terms of the solutions given in equation [3], as:

$$\begin{aligned} \mathrm{I} &= < A^*_{||}(z)A_{||}(z) > + < A^*_{\perp}(z)A_{\perp}(z) >, \\ \mathrm{Q} &= < A^*_{||}(z)A_{||}(z) > - < A^*_{\perp}(z)A_{\perp}(z) >, \\ \mathrm{U} &= 2\mathrm{Re} < \mathrm{A}^*_{||}(\mathrm{z})\mathrm{A}_{\perp}(\mathrm{z}) >, \\ \mathrm{V} &= 2\,\mathrm{Im} < \mathrm{A}^*_{||}(\mathrm{z})\mathrm{A}_{\perp}(\mathrm{z}) >, \end{aligned} \quad (4)$$

and use the same to find out the ellipticity angle χ and polarization angle ϕ from $\tan 2\chi = \frac{\mathrm{V}}{\sqrt{\mathrm{Q}^2+\mathrm{U}^2}}$, and $\tan 2\psi = \frac{\mathrm{U}}{\mathrm{Q}}$. Using eqns.[4] and [3] we have evaluated the same (χ and ϕ), in the limit $m_a \to 0$. Resulting ellipticity and polarization angles, after the ray has traveled distance z, turns out to be, $\chi \sim 0$, $\phi = \frac{(\mathcal{B}z)^2}{16M^2}$, in agreement with [5].

LIV Propagation of $A_{||}(z)$.

Next we have used eqn. [3] to find the phase (v_p) and group (v_g) velocities for $A_{\perp}(z)$ and $A_{||}(z)$. Our analysis shows that v_p and v_g for $A_{\perp}(z)$, remains unchanged but they undergo modifications for $A_{||}(z)$. The expression of v_p for $A_{||}(z)$, is given by:

$$\mathrm{v}_p = \frac{\left[\sqrt{\left(1-\frac{g_{c\gamma\gamma}B\sin\Theta}{\omega}\right)}+\sqrt{\left(1+\frac{g_{c\gamma\gamma}B\sin\Theta}{\omega}\right)}\right]}{2}. \quad (5)$$

One can check from eqn. [5] that v_P would become complex for $\omega < g_{\gamma\gamma c}\mathcal{B}sin\Theta = \omega_c$, and the corresponding wave would get attenuated. Furthermore, it can easily be verified by expanding the square-root in binomial series, that for $\omega > \omega_c$, v_p for$A_{||}(z)$ is subluminal.

The situation however doesn't remain the same for v_g of $A_{||}(z)$. Even though for some low values of $\omega > \omega_c$, v_g can become superluminal, however at very large values of ω it remains luminal. A plot of the behavior of the same is provided in Fig.[1]. As can be seen from the plot that with increasing energy, the group velocity v_g approaches that of light.

We have numerically estimated the time delay in signal propagation per km ($\frac{\delta t}{km}$) between the $||$ and $\perp$ modes in a magnetized vacuum with field strength B = 4.4 × 10^{13} Gauss and axion photon coupling constant $g_{a\gamma\gamma} = 10^{-11}\mathrm{GeV}^{-1}$. For energies around 0.82 × 10^{-15} GeV the delay per km turns out to be 0.004 × 10^{-5} sec. Observing this delay in laboratory conditions is difficult however, it may not be the same in astrophysical situations. For instance, in the case of magnetars with magnetospheric radius equal to 100 km, this delay would translate to 10^{-6} sec. With the available current technology for time resolution, this is an achievable precision.

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