Shear viscosity of neutron star matter in the presence of strong magnetic field

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Introduction

Shear viscosity plays an important role in neutron star physics [1, 2]. It can be crucial in the relaxation of hydrodynamic motions such as differential rotation within a star and stellar oscillations. It can also be important in the study of pulsar glitches and free precession of neutron stars [3]. The first extensive calculation of transport coefficients of non-magnetic dense matter especially for neutron stars was done by Flowers and Itoh [4]. Thereafter several authors have studied transport properties of non-magnetic stellar matter, only few studies [5–7] exist for magnetic case.

Neutron stars can have large magnetic field ($\sim 10^{12}$ G). Discovery of magnetars (magnetic field varies from $10^{14} - 10^{15}$G at the surface to $\sim 10^{18}$ G at the core) enhanced the interest of the study of neutron star magnetic field [8]. Such high magnetic fields may have significant effects on the transport properties of the neutron star crust as well as the core. We investigate for the first time the effect of strong magnetic field on the shear viscosity of the dense core of neutron star matter.

The viscous pressure tensor of the quark-gluon plasma in a strong magnetic field has been illustrated recently [9]. The viscous pressure tensor of magneto-active plasma is azimuthally anisotropic and characterised by seven viscosity coefficients, among which five are shear viscosities and two are bulk ones. They consider a simple model based on the non-relativistic approximation in which the transverse fluid flow is governed by the Navier-Stokes equations. It was shown that four of shear viscosity coefficients vanish when the field strength is much larger than the plasma temperature squared [9].

Viscosity in a strong magnetic field

To calculate the transport coefficients described above one has to solve the following coupled transport equations. The kinetic equation for the distribution function f

$$p_k^\mu \partial_\mu f_k(x,p_k) = q_k F^{\mu\nu}(x)\frac{\partial f_k(x,p_k)}{\partial p_\mu} p_\nu + \mathcal{C}_{kl}(x,p_k) \quad (1)$$

where f_k and q_k are the equilibrium distribution function and electric charge, respectively for the species k and $\mathcal{C}_{kl}$ is the collision integral.

The equilibrium distribution function is

$$f_k^0 = \frac{g_s/h^3}{e^{\frac{-\mu_E + v^\alpha p_\alpha}{k_B T}} \pm 1} \quad (2)$$

As the right hand side of the above equation vanishes at equilibrium, the kinetic equation can be written as an equation for ∂f, which indicates the deviation from equilibrium. So, the kinetic equation can be expressed as

$$p_k^\mu \partial_\mu f_k^0 = q_k \ F^{\mu\nu}(x,p_k)\frac{\partial(\partial f_k)}{\partial p_\mu} p_\nu + \mathcal{C}_{kl}[\partial f_k]. \quad (3)$$

The viscous pressure generated by a deviation from equilibrium is given by the tensor $\Pi^{\alpha\beta} = -\int \frac{d^3p}{E_p} p^\alpha p^\beta \partial f$. This can be expressed in terms of shear viscosity coefficients as $\Pi^{\alpha\beta} = \sum_{n=0}^{4} \eta_n V^{(n)}_{\alpha\beta}$. To calculate shear viscosities we sought the solution to the kinetic equation in the form

$$\partial f_k = \sum_{n=0}^{4} g^{(n)} \ V^{(n)}_{\alpha\beta} v_k^\alpha v_k^\beta \ . \quad (4)$$

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To calculate the $g^{(n)}$'s which are functions of p_k^2, we have to solve the kinetic equation. Then the shear viscosity coefficients for the species k are determined as

$$\eta_n \;=\; -\frac{2}{15}\int p_k^4 g_k^{(n)} d^3p_k/\epsilon_k^3. \qquad (5)$$

Viscosity of collision-less matter

In strong magnetic field, we assume that the deviation from equilibrium due to the strong magnetic field is much larger than due to the particle collisions. In this case, g_n can be determined by substituting ∂f into the kinetic equation (Eqn. 4) and equating the co-efficients of various tensors $V_{\alpha\beta}^{(n)}$ on the both sides of the equation. This is solved by successive approximation in power of $1/\omega_B$, where synchrotron frequency $\omega = Z\ e\ B/m\ c^2 = Z\ e\ B/\epsilon$. To a first approximation, we completely neglect the collision integral. This leads to $g_1 = g_2 = 0$ & $g_4 = 2\ g_3 = -\frac{\epsilon^2}{2\ T\ Z\ e\ B}\ f_0(1-f_0)$ leaving the non-zero components of shear viscosity as

$$\eta_4 = 2\ \eta_3 = \frac{4}{15}\int \frac{\epsilon^3\ v^4}{4\ T\ Z\ e\ B} f_0(1-f_0)d^3p \qquad (6)$$

In the neutron star core all the integrals (integral in the above equation as well as collision integrals) are greatly simplified because all the particles are strongly degenerate so that the colliding particles can be placed on their Fermi surfaces. In the degenerate case, shear viscosity coefficients turn out to be $\eta_4 = 2\ \eta_3 = \frac{4}{5}\frac{p_F^2\ n}{Z\ e\ B}$.

Contribution of collisions

Next we consider the effect of collisions. We assume the relaxation time to be independent of energy, this is know as relaxation time approximation. Under this approximation, we can write the collision integral as $\mathcal{C}[\partial f] = -\nu\partial f$, where ν is effective collision rate. In the strong field limit $\omega_B >> \nu$. After some intriguing algebra we arrive at $g_1 = g_2/4 = -\frac{\epsilon^3\nu}{8\ T\ (Z\ e\ B)^2}\ f_0(1-f_0)$ that on the other hand leads to

$$\eta_1 = \eta_2/4 = \frac{3}{80}\frac{n^3\sigma_t\pi^2}{(Z\ e\ B)^2} \qquad (7)$$

where σ_t is the transverse cross-section and is given by $\nu = n\ v\ \sigma_t$.

Summary

We study the effect of strong magnetic field on the transport properties of neutron star matter. The study of shear viscosity of dense matter is important to understand several astrophysical phenomena. Besides the role of shear viscosity in damping the r-mode instability as well as in pulsar glitches and free precession of neutron stars, it has an important contribution in the nucleation rate of bubbles in a first order phase transitions. We have earlier seen that the shear viscosity plays significant role in the initial growth rate of a bubble [2, 10]. This needs further study in connection with phase transition to any exotic phase such as hyperon or Bose-Einstein condensation of antikaons. in neutron stars.

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