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The Hoyle state in nuclear Lattice EFT

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The "Hoyle state" is an excited state of the ^{12}C nucleus with quantum numbers $J^p = 0^+$, slightly above the ^{8}Be–α threshold. In red giant stars, ^{12}C is generated via the so-called triple-alpha process, whereby three ^{4}He nuclei (α particles) fuse to form ^{12}C. However, as the triple-alpha process cannot, by itself, explain the observed abundance of ^{12}C in the Universe, Hoyle postulated in 1954 the existence of an excited 0^+ state of ^{12}C, at a very specific energy above the ^{8}Be–α threshold. Such resonant enhancement could then provide a sufficiently high rate of production of ^{12}C to account for the observed abundance [1]. Soon afterwards, the predicted state was detected at Caltech [2, 3], and the modern value for its energy is $\varepsilon = 379.47(18)$ keV above the 3α threshold, while the total and radiative widths are $\Gamma_{\rm tot} = 8.3(1.0)$ eV and $\Gamma_\gamma = 3.7(5)$ meV.

The effects of changes in ε on the synthesis of the life-essential elements carbon and oxygen in red giant stars has been investigated numerically in terms of highly sophisticated stellar evolution models. Livio *et al.* [4] modified the value of ε by hand and studied the triple-alpha process in the core and shell He burning up to the asymptotic giant branch stage in the stellar evolution. These calculations have been refined by Oberhummer *et al.*, who concluded that the production of either ^{12}C or ^{16}O becomes strongly suppressed for changes larger than $\delta(\varepsilon) \simeq \pm 100$ keV in the position of the Hoyle state [5, 6]. In essence, if ε is lowered too much, the triple-alpha process ignites at a significantly lower stellar core temperature, and hence little energy is available for the process $^{12}\mathrm{C} + {}^4\mathrm{He} \to {}^{16}\mathrm{O} + \gamma$. Conversely, if ε is raised too much, the triple-alpha process ignites at a much higher core temperature, and hence most of the ^{12}C formed is immediately converted into ^{16}O and ^{20}Ne already before the conclusion of core He burning. However, since a change of ±100 keV in ε could still be tolerated, which is a $\sim 25\%$ modification, the fine-tuning was revealed to be less severe than originally believed [7].

Nevertheless, the translation of these astrophysical findings into anthropic constraints on fundamental parameters requires a direct connection of $\delta(\varepsilon)$ to the fundamental theory of the strong interactions, Quantum Chromodynamics (QCD) and its fundamental parameters, the light quark masses m_q. We address this question by means of an *ab initio* calculation of the sensitivity of ε to changes in m_q and the electromagnetic (EM) fine structure constant $\alpha_{\rm em}$. For this purpose, we carry out large-scale numerical lattice calculations for the energies and energy differences relevant to the triple-alpha process within the framework of chiral Effective Field Theory (EFT). The discretized (lattice) version of chiral EFT was formulated in Ref. [8] (see Ref. [9] for a recent review). We have successfully applied this novel approach to the spectra and properties of light nuclei [10], to dilute neutron

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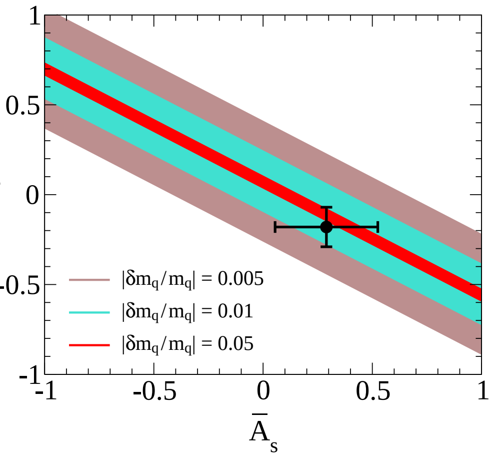

FIG. 1: "Survivability bands" for carbon-oxygen based life from Eq. (1), due to 0.5% (broad outer band), 1% (medium band) and 5% (narrow inner band) changes in m_q in terms of $\bar{A}_s$ and $\bar{A}_t$ (datapoint from N^{2}LO analysis of Ref. [14]).

matter [11], and to the structure of the Hoyle state [12]. Our findings for the triple-alpha process have been reported in Ref. [13].

From our nuclear Lattice EFT results, we find that the condition $|\delta(\varepsilon)| < 100$ keV corresponds to a predicted tolerance $|\delta\alpha_{\rm em}/\alpha_{\rm em}| \simeq 2.5\%$ of carbon-oxygen based life to shifts in $\alpha_{\rm em}$. This result is compatible with the $\simeq 4\%$ bound reported by Ref. [6]. For shifts in m_q, we find

$$\left| \Big[0.572(19)\,\bar{A}_s + 0.933(15)\,\bar{A}_t - 0.064(6) \Big] \times \left(\frac{\delta m_q}{m_q} \right) \right| < 0.15\%, \qquad (1)$$

in terms of the slopes of the inverse scattering lengths a_s^{-1} and a_t^{-1} at the physical pion mass,

$$\bar{A}_s \equiv \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\rm ph}}, \qquad \bar{A}_t \equiv \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\rm ph}}, \qquad (2)$$

where the subscripts s and t denote the spin–0 ($^1S_0$) and spin–1 ($^3S_1$) NN partial waves. The most up-to-date N^{2}LO analysis of Ref. [14] gives $\bar{A}_s \simeq 0.29^{+0.25}_{-0.23}$ and $\bar{A}_t \simeq -0.18^{+0.10}_{-0.10}$, shown in Fig. 1, superimposed on the Lattice EFT bounds. In spite of the relatively large uncertainties, Fig. 1 shows that our Universe is not extremely fine-tuned with respect to the Hoyle state. The uncertainties in $\bar{A}_s$ and $\bar{A}_t$ will be significantly reduced by upcoming Lattice QCD calculations.

References

[1] F. Hoyle, Astrophys. J. Suppl. Ser. **1**, 121 (1954).

[2] D. N. F. Dunbar, R. E. Pixley, W. A. Wenzel, and W. Whaling, Phys. Rev. **92**, 649 (1953).

[3] C. W. Cook, W. A. Fowler, C. C. Lauritsen, and T. Lauritsen, Phys. Rev. **107**, 508 (1957).

[4] M. Livio, D. Hollowell, A. Weiss, and J. W. Truran, Nature **340**, 281 (1989).

[5] H. Oberhummer, A. Csótó, and H. Schlattl, Science **289**, 88 (2000).

[6] H. Oberhummer, A. Csótó, and H. Schlattl, Nucl. Phys. A **689**, 269 (2001); H. Schlattl *et al.*, Astrophys. Space Sci. **291**, 27 (2004).

[7] S. Weinberg, "Facing Up" (Harvard University Press, Cambridge, Massachusetts, 2001).

[8] B. Borasoy, E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, Eur. Phys. J. A **31**, 105 (2007).

[9] D. Lee, Prog. Part. Nucl. Phys. **63**, 117 (2009).

[10] E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, Eur. Phys. J. A **41**, 125 (2009); Phys. Rev. Lett. **104**, 142501 (2010); Eur. Phys. J. A **45**, 335 (2010); Phys. Rev. Lett. **106**, 192501 (2011).

[11] E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, Eur. Phys. J. A **40**, 199 (2009).

[12] E. Epelbaum, H. Krebs, T. A. Lähde, D. Lee, and U.-G. Meißner, Phys. Rev. Lett. **109**, 252501 (2012).

[13] E. Epelbaum, H. Krebs, T. A. Lähde, D. Lee, and U.-G. Meißner, Phys. Rev. Lett. **110**, 112502 (2013); Eur. Phys. J. A **49**, 82 (2013).

[14] J. C. Berengut *et al.*, Phys. Rev. D **87**, 085018 (2013).