

## Opportunities and problems in determining proton and light nuclear radii

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### Introduction

The advances in experimental techniques in recent years has provided a remarkable improvement in the accuracy of measurements in atomic physics. The 1S-2S interval in hydrogen for example is measured to an accuracy of 2 parts in  $10^{14}$ . The hydrogen atom as such has been a precision tool for testing predictions of quantum electrodynamics (QED) which (expressed as powers of  $\alpha$ , the fine structure constant) are calculated through  $\mathcal{O}(\alpha^8)$ . With experiments and QED corrections becoming more and more accurate, the difference between theory and experiment is dominated by the nuclear structure corrections. The above, in principle, provides an opportunity to determine the proton radius or even light nuclear radii from precise measurements of atomic spectra in contrast to the determinations from the less accurate electron-nuclei scattering experiments. A recent example is that of the Lamb shift measurement in muonic hydrogen using pulsed laser spectroscopy. A precise value for the transition energy,  $\Delta = E_{2P_{3/2}}^{f=2} - E_{2S_{1/2}}^{f=1} = 206.2949(32)$  meV, in muonic hydrogen was given [1]. Comparing the measured value with a model calculation of this difference which included the proton structure effects, a new (accurate) value of the proton radius, namely,  $r_p = 0.84184(67)$  fm was published.

### The Shrinking Proton

The radius published in [1] was smaller than the CODATA [2] value of 0.8768(69) fm which is an average of several scattering and hydro-

gen spectroscopy data. This difference taken together with the claim of high precision gave rise not only to a vast amount of literature on the proton radius calculations but also to popular articles ranging from scientific magazines with titles like “The incredible shrinking proton that could rattle the world” to New York Times reporting “For a proton, a little off the top (or side) could be big trouble”. The authors in [1] mentioned that the new value of the radius,  $r_p = 0.84184(67)$  fm either implied that the Rydberg constant has to be shifted by -110 kHz/c or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient. Ref. [3] even discussed the possible role of the proton in redefining the fundamental constants and the international system of units. The problem which came to be known as the “proton radius puzzle” was reinforced recently by the same team [4] with a new accurate value of  $r_p = 0.84087(39)$  fm.

### Finite size effects

In the midst of all the accurate claims and the worries which followed one should not forget that any “accurate” measurement in atomic physics is masked by the uncertainty introduced due to the nuclear structure. In the case of the muonic hydrogen atom, we found [5] that the uncertainty not only arises from the uncertainty in the measured electromagnetic proton form factors but also from the approach used to incorporate the effects. In [1] the energy shift was calculated using a Coulomb potential modified to take into account the proton charge distribution. This expression, namely,  $\Delta(= E_{2P_{3/2}}^{f=2} - E_{2S_{1/2}}^{f=1}) = 209.9779(49) - 5.2262r_p^2 + 0.0347r_p^3$  meV, was then compared with the experimental value to determine the proton radius.

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A sophisticated modification of the Coulomb potential is offered by quantum field theory (QFT). In a QFT description of a muon-proton amplitude [6], the coupling of the photon to a point-like particle (vertex),  $e\gamma_\mu$ , has to be replaced by  $e(F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu}q^\nu/2m_p)$  where  $q_\mu$  is the four momentum carried by the photon,  $q^2 = q_\mu q^\mu$  and  $F_i(q^2)$  are the electromagnetic form-factors which encode the information on the structure of the proton. Such an approach led us to find  $\Delta(= E_{2P_{3/2}}^{f=2} - E_{2S_{1/2}}^{f=1}) = 209.16073 + 0.1139r_p - 4.3029r_p^2 + 0.02059r_p^3$  meV and hence a radius of  $r_p = 0.831$  fm. We could conclude that the proton shrunk further but it would rather be appropriate to mention that the accuracy is dependent on the approach used for the finite size effects.

In order to resolve the issue further, to check for missing QED effects, there are experiments planned for similar measurements with laser

spectroscopy for muonic deuterium and electronic as well as muonic helium ions. The present talk will discuss the theoretical aspects of the nuclear finite size effects in atoms and the general status of the so called proton radius puzzle.

## References

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