Deformation in Λ-hypernuclei

Bipasha Bhowmick,* Abhijit Bhattacharyya, and G. Gangopadhyay
Department of Physics, University of Calcutta
92, A. P. C. Road, Kolkata-700 009, India

One of the interesting aspects of hypernuclei is the structural change caused by the hyperon. As an impurity in normal nuclei, a hyperon may be expected to induce many effects on the core nucleus, such as change in size, shape change, modification of its cluster structure, occurrence of nucleon and hyperon skin or halo, shift of neutron drip line to more neutron-rich side etc. Some of those have been observed in light p-shell hypernuclei [1, 2].

To perform a systematic and quantitative study of the structure change of the p and sd-shell nuclei, caused by the hyperon, we study the binding energy, quadrupole deformation, and the root mean square radii of a number of hypernuclei within a relativistic mean field (RMF) model namely FSUGold [3]. This parametrization has already been extended to include hyperons and to study the properties of hypernuclear systems[4]. In the present work, we limit the type of deformation to azimuthally symmetric and reflection symmetric systems which corresponds to prolate and oblate ellipsoids for quadrupole deformation. The method of solution has been explained in detail in Ref. [5]. To calculate the energy surface as a function of quadrupole deformation, we have used the method of Lagrange's undetermined multiplier. Thus the deformation constrained Lagrangian density has been written as

$$\mathcal{L}' = \mathcal{L} - \lambda \hat{Q}_D \bar{\Psi}\Psi, \qquad (1)$$

where $\hat{Q}_D = f(r)(3z^2 - r^2)$ is the quadrupole operator multiplied by a radial damping function $f(r)$. For each Q_D, we have calculated the quadrupole deformation parameter β from the charge quadrupole moment using the relation

$$Q_p = \sqrt{\frac{16\pi}{5}}\frac{3}{4\pi} Z R_0^2 \beta \qquad (2)$$

where $R_0 = 1.2A^{1/3}$ fm.

The proton rms radius is defined as

$$r_p = \sqrt{\frac{1}{Z}\int r^2 \rho_p d\mathbf{r}} \qquad (3)$$

Here ρ_p is the proton density. Neutron rms radius (r_n) and nuclear rms radius (r) are calculated in an analogous way. The calculations have been performed for hypernulei $A < 30$ and the results are shown in Table 1.

In our calculation, the ground state of Λ is a pure s-state. The first excited state is a negative parity state which has contributions from both the p orbitals. We thus indicate the solutions for hypernuclei with the hyperon in the ground or the first excited states with 's' and 'p' in parentheses in the table. We have looked for both prolate and oblate minima in normal nuclei and hypernuclei. It is observed that in all the cases, the deformed hypernuclear minimum corresponds only to prolate deformation. The lowest state in the p-shell, the $p_{1/2}$ state, splits up in $K^\pi = 1/2^-$ and $K^\pi = 3/2^-$ Nilsson states in presence of reflection symmetric axial deformation. For positive deformation, the former is the lowest energy state. Thus, in case of the excited state, the hyperon occupies the $K^\pi = 1/2^-$ state. No oblate minima are observed in the hypernuclear systems. Very little information is available for the Λ separation energy in the ground state of the hypernucleus; the existing experimental values agree reasonably well with our calculation.

From Table 1, we see that when the Λ is placed in the ground state, the deformation changes slightly on, with only one exception

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[*] Electronic address: `bips.gini@gmail.com`

in the case of $^{13}_{\Lambda}$C. The ground state of the core nucleus is generally prolate, with the exception of ^{12}C and ^{28}Si, the former of the two being oblate and the latter spherical. Except in these two nuclei, the deformation, on inclusion of a hyperon in the lowest energy state, may increase as well as decrease, but in general the shape remains similar to that of the core nucleus in its ground state.

TABLE I: Binding energy/nucleon(-E/A) and Λ separation energy (E_{Λ}) in MeV, quadrupole deformation parameter β, and the rms radius in fm at the minimum energy for the core nucleus and the corresponding hypernucleus in different Λ states.

Nucleus (Λ-state)	-E/A	E_{Λ}	β	r_p	r_n	r
^{8}Be	5.668	-	0.38	2.41	2.31	2.36
	5.533		-0.37	2.39	2.28	2.33
$^{9}_{\Lambda}$Be (s)	5.837	7. 189*	0.34	2.40	2.38	2.34
^{10}B	6.201	-	0.27	2.41	2.26	2.33
	6.116		-0.20	2.39	2.24	2.32
$^{11}_{\Lambda}$B (s)	6.501	9.501	0.33	2.54	2.34	2.40
^{12}C	7.175	-	-0.21	2.36	2.18	2.27
$^{13}_{\Lambda}$C (s)	7.534	11.842†	0.13	2.59	2.20	2.39
$^{13}_{\Lambda}$C (p)	6.683	0.779	0.14	2.61	3.00	2.92
^{18}F	7.562	-	0.14	2.70	2.45	2.58
	7.561		-0.11	2.69	2.44	2.57
$^{19}_{\Lambda}$F (s)	7.883	13.661	0.13	2.71	2.52	2.56
$^{19}_{\Lambda}$F (p)	7.399	4.465	0.14	2.71	2.62	2.63
^{22}Na	7.565	-	0.22	2.85	2.57	2.72
	7.475		-0.13	2.83	2.55	2.69
$^{23}_{\Lambda}$Na (s)	7.904	15.362	0.21	2.86	2.62	2.70
$^{23}_{\Lambda}$Na (p)	7.537	6.921	0.22	2.86	2.69	2.74
^{24}Mg	7.814	-	0.25	2.91	2.60	2.76
	7.640		-0.15	2.89	2.59	2.74
$^{25}_{\Lambda}$Mg (s)	8.142	16.014	0.23	2.91	2.65	2.74
$^{25}_{\Lambda}$Mg (p)	7.823	8.039	0.24	2.91	2.71	2.78
^{25}Mg	7.872	-	0.20	2.88	2.64	2.76
	7.793		-0.13	2.87	2.63	2.75
$^{26}_{\Lambda}$Mg (s)	8.248	17.648	0.22	2.92	2.63	2.75
$^{26}_{\Lambda}$Mg (p)	7.927	9.30	0.23	2.92	2.69	2.79
^{25}Al	7.657	-	0.15	2.95	2.56	2.77
	7.565		-0.11	2.94	2.55	2.76
$^{26}_{\Lambda}$Al (s)	8.001	16.601	0.14	2.95	2.61	2.75
$^{26}_{\Lambda}$Al (p)	7.676	12.103	0.15	2.95	2.67	2.79
^{26}Al	7.828	-	0.11	2.92	2.60	2.76
	7.795		-0.09	2.92	2.59	2.76
$^{27}_{\Lambda}$Al (s)	8.200	17.872	0.13	2.96	2.59	2.76
$^{27}_{\Lambda}$Al (p)	7.874	9.07	0.14	2.96	2.65	2.80
^{28}Si	8.080	-	0.00	2.95	2.54	2.77
$^{29}_{\Lambda}$Si (s)	8.453	18.897	0.06	3.00	2.54	2.78
$^{29}_{\Lambda}$Si (p)	8.118	9.182	0.06	3.00	2.60	2.82

However, in the case of $^{13}_{\Lambda}$C, the shape appears to change drastically on inclusion of Λ. The ground state of ^{12}C shows negative deformation *i.e.* oblate shape, whereas the corresponding hypernucleus $^{13}_{\Lambda}$C is prolate. This is consistent with the fact that the energy surface shows an oblate minimum in the core nuclei but not in the hypernuclei.

To summarise we have studied the deformation of core and hypernuclei in the RMF approach. The Λ binding energies agree reasonably well with the experimental values. The inclusion of a Λ hyperon changes the energy surface making it steeper. There exists no oblate minimum in any light hypernucleus. Results of the present calculation differ from the previous ones. The latter usually predict that inclusion of a Λ tends to drive the shape to spherical. Our results show that the change in β is usually small with possible exceptions. If the core nucleus is prolate, the deformation may increase in some cases on addition of a hyperon. In general, the nucleon density profile changes to a small extent on inclusion of the hyperon, whether in the ground state or the first excited state. When the Λ goes to the p-state, both the deformation and radius increases by a small amount. The only exception is $^{13}_{\Lambda}$C where, the hyperon being very loosely bound, creates a halo-like structure.

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