Nucleonic Magnetic Moment with Effective charge and Mass

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Introduction:

Hadronic physics describes the hadrons in term of their fundamental quarks and gluon degree of freedom. Even the constituent quarks explain the magnetic moment of nucleons effectively, the theoretical and experimental results show variation to an extent. Thus we attempt to calculate the magnetic moment of the nucleons taking the effective mass and screen charge [1,2]. Whenever a quark is probed by a soft photon, the mass and charge of nucleon get modified because of the quarks interactions with the spectator quarks inside the baryons. As the quarks are deeply confined inside the baryon, soft photon see a coherent internal structure of baryon. Thus, the mass and charge parameter appearing in the magnetic moment must reflect the effect of confinement. If we consider, hadrons to be the bound state of the interacting quarks, the mass difference between the multiplets within the group can be attributed to a spin-spin interaction. QCD suggests, color quarks to interact through the exchange of gluons even being confined with the bound state. As a result the mass splitting of light hadrons can be described in the frame work of non- relativistic quark model. The effective potential for the interaction of two quarks is written as $V_{ij} = \pm\alpha_s(\frac{\lambda_i}{2}\frac{\lambda_j}{2})\left(-\frac{2\pi}{3}\frac{\sigma_i\sigma_j}{m_i m_j}\delta(r_{ij})\right)$ where α_s represents strong coupling constant, and λ represents the Gell-Mann matrices, whereas $\pm$ sign represents the interaction of two quarks or quark and anti-quark respectively. If there is no color charge then the potential V_{ij} between two quarks have negative value. The remaining part of the interaction is due to gluon exchange and gives the contribution to the potential which binds the quarks.

Screen Mass

The spin-spin interactions give rise to the hyperfine splitting as

$$\Delta E_{hfs} = \frac{8\pi\alpha}{3}|\psi(0)\rangle^2 \frac{s_i.s_j}{m_i m_j}.$$

The term $|\psi(0)\rangle^2$ is considered as the phenomenological constant and represents the value of the wave function $\psi(r_1,r_2)$ at origin with zero separation, α represents the fine structure constant. To get the result in the QCD, $\alpha = \frac{2}{3}\alpha_s$ for baryons where $\alpha_s = 4\pi$ .Therefore, the hyperfine splitting value for the baryons is given as $\Delta E_{hfs} = \frac{16\pi}{9}\alpha_s|\psi(0)\rangle^2 \sum_{i<j}\frac{s_i.s_j}{m_i m_j}$. In this simple model of hadrons, masses are supposed to arise from the sum of the constituent quark masses and hyperfine interactions. Thus, for the baryons the masses of the S-wave 56-plet [3] the expression is given as $M_B = \sum_{i=1}^{3} m_i + \sum_{i<j} b_{ij}s_i.s_j$ and $\Delta M = \sum_{i<j} b_{ij}s_i.s_j$, $b_{ij} = \frac{16\pi}{9m_i m_j}\alpha_S|\psi(0)\rangle^2$, $s_i.s_j$ represents the effective spin of the three quarks state for baryons and term $m_i m_j$ represents the effective mass. where α , b_{ij} and quark masses are free parameters and explains the hadron mass spectrum with a constituent set of values as suggested by Gell-Mann Okubo mass formula [4]. Therefore, for the baryons in constituent quark model the spin term can be solved as $\sum_{i<j} S_i.S_j = -\frac{3}{4}$ This gives $M_B = m_1 + m_2 + m_3 + \frac{1}{2}\sum_{i<j} b_{ij}s_i.s_j$. The factor ½ here introduces or reflects the reduction in the strength of the gluon exchange between the quarks in a color anti- triplet state relative to that between a quark and anti-quark in color singlet state. The flavor symmetry implies $m_1 = m_2$ so $b_{23} = b_{13}$ giving $M_B = 2m_1 + m_3 + \frac{1}{2}[b_{12}s_1.s_2 + b_{13}[s_1.s_3 + s_2.s_3]]$. On substituting the value of the S, s_1, s_2, s_3 the effective mass formula for the baryons is given by $m_1^{eff} = m_2^{eff} = m + \alpha b_{12} + \beta b_{13}$ and $m_3^{eff} = m + 2\beta b_{13}$ where α and β comes out to be $\alpha = \frac{1}{8}, \beta = -\frac{1}{4}$ Therefore, we get effective mass for nucleons as $M_N = 3m_u - \frac{3}{4}b_{uu}$. The values of m_u, m_s and are obtained as $m_u = m_d = 363MeV, m_s = 538MeV,$

$$\frac{\alpha}{m_u^2} = b_{uu} = b_{dd} = b_{ud} = 200MeV .$$

Screened charge

Due to the gluon self-coupling interactions, the vacuum is filled with the virtual particles i.e. virtual gluon pairs. The gluon color seems to be leaked out in the vacuum as a consequence penetrating probe(photon) will feel an effective charge that become smaller with the smaller distance. Therefore, the effective small coupling becomes small at smaller distances. This effect shield the color charge of the quark core, As a result, the net color charge decreases as we penetrate through the virtual gluon cloud. Thus if the quarks are make to collide at very high energies they will penetrate further the virtual cloud of the color charge and experience a reduction in the color charge. In the symmetric constituent quark model, the magnetic moment of nucleon is given as

$\mu_N(aab) = \frac{4}{3}\frac{e_a}{2m_a} - \frac{1}{3}\frac{e_b}{2m_b}$

Therefore, the effective charge of the quark a is represented by e_α^N i.e. $e_a^N = e_a + \alpha_{ab}e_b + \alpha_{ac}e_c$ and the isospin symmetry implies $\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta$ and

Modified Nucleon magnetic moment:

Magnetic moment operator is given as $\mu = \sum_i \frac{e_i}{2m_i}$ and in terms of shielding mass it becomes $\mu_N = \sum_i \frac{e_i}{2m_i^{eff}}$

where $i = 1, 2, 3$.Using above mentioned values of m_i^{eff} the values of the effective masses of the constituents are given as in table 1:

Particle constituent effective mass	Calculated Mass (MeV)	Ref. Mass (MeV) [5]
m_u^p	338	338
m_d^p	263	265
m_u^n	263	265
m_d^n	338	338

$\mu_n = \frac{4(-1+\beta)m_n}{9\left(\frac{b_{dd}}{8} - \frac{b_{ud}}{4} + m_d\right)} - \frac{2(1-\beta)m_n}{9\left(\frac{-b_{ud}}{2} + m_u\right)}$.In the above equation α, β and γ are the parameters. For SU(3) symmetry,

$m_u = m_d = m_n = 336MeV, m_s = 450MeV$

$b_{uu} = b_{ud} = b_{dd} = 200MeV$

The value of shielding effect comes out to be $\alpha = 0.15$

$\alpha_{us} = \alpha_{ds} = \alpha$ So screened charge of quarks is given as $e_u^p = \frac{2}{3}\left(1 + \frac{1}{2}\beta\right)$ $e_d^p = -\frac{1}{3}(1-4\beta)$ and of neutron are $e_u^N = \frac{2}{3}(1-\beta)$ and $e_d^N = -\frac{1}{3}(1-\beta)$

Magnetic moment in terms of screen charge is given $\mu_N = \sum_i \frac{e_i^{eff}}{2m_i}$ Using the screen charged equation parameters and modified baryon magnetic momentum operator for shielding charge, we get on substituting β=0 in case of SU(6) symmetry, $\mu_P = 1$, $\mu_N = -\frac{2}{3}$ The magnetic moment operator in terms of the both effective mass and charge scheme i.e. $\mu_N = \sum_i \frac{e_i^{eff}}{2m_i^{eff}}\sigma_i$, the magnetic moment terms for nucleons is given as[6]

$\mu_P = -\frac{\left[(-1+4\beta)m_n\right]}{9\left(\frac{-b_{ud}}{2} + m_d\right)} + \frac{8\left(1+\frac{\beta}{2}\right)m_n}{9\left(-\frac{b_{ud}}{4} + \frac{b_{uu}}{8} + m_u\right)}$

Table 2 Comparisons of magnetic moment of baryons in the different schemes:

Baryon particles	Effective mass (μ_N)	Shielding charge (μ_N)	Effective mass + Shielding charge (μ_N)	Experimental values (μ_N) [7]
p	2.863	2.79	1.1	2.79
n	-2.20	-1.91	-0.64	-1.91

Conclusions

We compare the magnetic moment of nucleons in different schemes i.e. effective mass, shielding or screen charge, combine effect of effective mass and shielding charge. From this comparison we conclude that, in each case the magnetic moment comes out to be approximately equal to that of the experimental values but in case of the combined effect of the effective mass and shielding charge, the magnetic moments are not corresponding to the experimental values. The possible justification for this mismatch could be because of the simultaneous application of both effects, and can better be studied individually to see their effects on the baryonic system. With a system having quarks and gluons, there may be present various interactions between the quarks through the exchange of gluons, that may lead to the mismatch between experiments and theory. Hence, we speculate some other effects to be dominant which needs to be studied to nullify the effects to match the result well.

Acknowledgement

This work has been partially supported by university grant commission Government of India via SR/41-959/2012.

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