Analysis of Λ-binding energies in the relativistic and non-relativistic approach

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Introduction

The Λ-hypernuclear systems have been analyzed by means of both relativistic and non-relativistic approaches. Relativistic approach takes into account the spin-orbit force occurring naturally in the theory [1] and also plays an important role in nuclear saturation phenomena [2]. Hence, in relativistic calculations, a good reproduction of the Λ-binding energies (B_Λ) is expected. Here, we make a comparative study of the B_Λ in hypernuclei using our non-relativistic approach [3] and the relativistic approach followed by Koutroulos and Grypeos [4].

Formulation

In our non-relativistic phenomenological approach, we have obtained a semi-empirical formula for B_Λ using the point nucleon (N) density $\rho_N(r)$ as an average of point proton density $\rho_p(r)$ and point neutron density $\rho_n(r)$:

$$\rho_N(r) = \frac{Z}{A_c}\rho_p(r) + \frac{N}{A_c}\rho_n(r). \quad (1)$$

The single-particle Λ-nucleus potential is obtained by folding zero-range ΛN potential with the point nucleon density of the core nucleus. Solving the eigenvalue equation for B_Λ, in the approximation $e^{-R/a} \ll 1$, leads to the following semi-empirical formula [3]:

$$B_\Lambda = D_\Lambda - \frac{\hbar^2\pi^2}{2\mu_{\Lambda A}}\left\{C_0' A_c^{-2/3} - C_1' A_c^{-1} + \cdots\right\}, \quad (2)$$

where the parameters are defined in ref. [3].

In the relativistic approach [4], the average local Λ-nucleus potential is constructed by means of an attractive scalar relativistic single particle potential $U_s(r)$ and a repulsive relativistic single particle potential $U_v(r)$ which is the fourth component of a vector potential. Writing the eigenvalue equation, in a way analogous to that of the non-relativistic case and solving for B_Λ (for heavy hypernuclei), the following expression [4] is obtained:

$$B_\Lambda^{(0)} = \frac{\mu c^2}{\lambda}\{1 + \lambda D_+(2\mu c^2)^{-1}\}\{1 - \left[1 + 2\lambda(\mu c^2)^{-1}\left(\frac{\hbar^2\pi^2\lambda}{2\mu R^2} - D_+\right)(1 + \lambda D_+(2\mu c^2)^{-1})^{-2}\right]^{1/2}\}, \quad (3)$$

where the symbols are defined in ref. [4].

Retaining the first term in the expansion of arctanx in powers of x, an improved form of B_Λ is obtained [4], which is given as

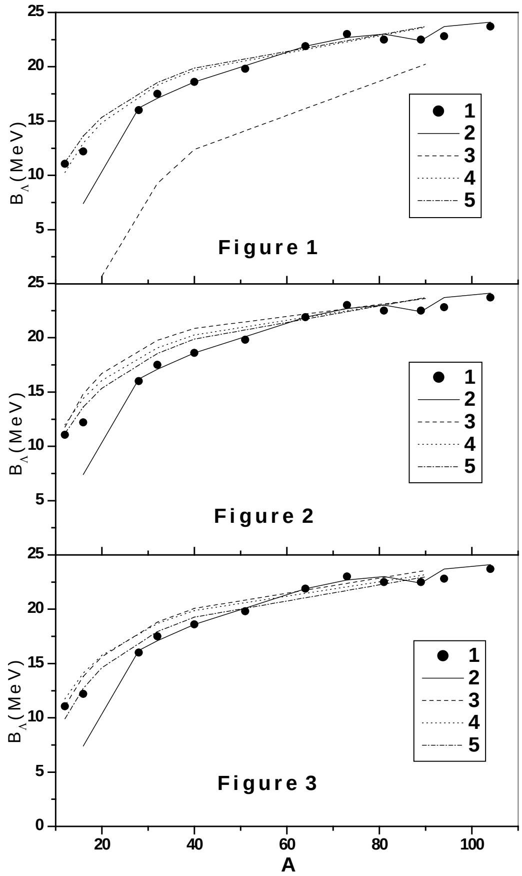
$$B_\Lambda^{(1)} = D_+ - \frac{\hbar^2\pi^2}{2\mu g\left(1+(\tilde{f}\eta_0 R)^{-1}\right)^2 R^2}, \quad (4)$$

where g, $\tilde{f}$ and η_0, defined in ref. [4], depend upon B_Λ but their values are estimated by using an approximate expression $B_{appr.} = D_+$ for B_Λ.

Result and Discussion

The radius and diffuseness parameters of the average nucleon density $\rho_N(r)$ are obtained [3] from the least square fit to eq. (1) for nuclei over a large mass number range. With these parameters, the χ^2 fit to the ground state B_Λ values of ${}^{28}_{\Lambda}Si$, ${}^{32}_{\Lambda}S$, ${}^{40}_{\Lambda}Ca$, ${}^{51}_{\Lambda}V$ and ${}^{89}_{\Lambda}Y$ is carried out [3] using eq. (2). The best fit value of D_Λ is 29.47 MeV. These parameters are then used to predict [3] the B_Λ values of ${}^{16}_{\Lambda}O$ and the heavy and spallation hypernuclei corresponding to the mass number range A = 64, 73, 81, 94 and 104. The experimental B_Λ data of ${}^{13}_{\Lambda}C$, ${}^{16}_{\Lambda}O$, ${}^{28}_{\Lambda}Si$, ${}^{32}_{\Lambda}S$, ${}^{40}_{\Lambda}Ca$, ${}^{51}_{\Lambda}V$, ${}^{89}_{\Lambda}Y$ and the upper limits of B_Λ in the case of the above mentioned mass number range, are shown as 1 in all the given figures. While our calculated B_Λ values obtained from fitting [3], along with the predicted values, are represented as 2 in all figures.

The B_Λ values calculated in ref. [4], are shown in the figures for A = 12, 16, 20, 32, 40 and 90. The calculated B_Λ values [4] for A = 140 and 208 are excluded from the plots as their experimental values are unavailable. The $B_\Lambda^{(0)}$, $B_\Lambda^{(1)}$ and B_Λ (exact) values, in ref.[4], with D_- = 443 MeV, D_+ = 30.77 MeV and r_0 = 1.022 fm, are plotted as 3, 4 and 5 in Fig. 1. These values of B_Λ's, further calculated in ref. [4], using D_- = 443 MeV and the corresponding best fit values of D_+ and r_0, are plotted in Fig. 2, as 3, 4 and 5.



In Fig. 3, the best fit values [4] of $B_\Lambda^{(1)}$ for D_- = 412.84 MeV, D_+ = 29.57 MeV and r_0 = 1.132 fm are plotted as 3. The $B_\Lambda^{(1)}$ values [4] with D_- = 443 (fixed) MeV, D_+ = 29.03 MeV, r_0 = 1.147 fm and m^*= 0.788 m are plotted as 4. The calculated values [4] of $B_\Lambda^{(1)}$, with the parameters D_- = 590.15 MeV, D_+ = 29.50 MeV, r_0 = 1.123 fm, m^*= 0.722m, obtained by least square fitting of experimental data, are plotted as 5 in Fig. 3.

From Fig. 1, we can see that the $B_\Lambda^{(0)}$ values (shown as 3), are way-off from the experimental data, while the $B_\Lambda^{(1)}$ and B_Λ (exact) values (shown as 4 and 5), are quite close to each other but differ slightly from the experimental data for comparatively lower mass numbers. The B_Λ values obtained by us [3] (shown as 2 in all the figures) give a fairly good account of the experimental data over a wide range of mass numbers. The difference in the predicted B_Λ of $^{16}_{\Lambda}O$ is not surprising as our semi-empirical formula for B_Λ is valid for heavy hypernuclei. The best fit values of $B_\Lambda^{(0)}$, $B_\Lambda^{(1)}$ as well as B_Λ (exact) in Fig. 2, (represented as 3, 4 and 5), are comparatively not so good for A< 60.

The $B_\Lambda^{(0)}$ values (shown as 3) are quite unrealistic in Fig. 1 and show considerable deviation from the experimental data in Fig. 2. The $B_\Lambda^{(1)}$ values (shown as 5), in Fig. 3, are comparatively better, while the others (shown as 3 and 4), are more or less same but show deviations from the experimental data for lower mass number range. The D_- parameter seems to play a significant role in the fitting. With the higher value of D_- and only marginal changes in other parameters, the B_Λ values are reproduced fairly well (shown as 5) in Fig. 3.

In comparison to the relativistic case [4] our non-relativistic results [3] give a much better reproduction of the experimental data, as is evident from Figs. 1, 2 and 3. This anomaly might be due to the approximations chosen in relativistic semi-empirical mass formula [4] for B_Λ of heavy hypernuclei. However, more information is needed to draw any definite conclusion about the significance of relativistic approach for the determination of B_Λ.

References

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