^{232}Th, a rigid rotor

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^{232}Th is a well deformed nucleus having asymmetric parameter $\gamma = 10^0$. The experimental energy gap $E3_1^+ - (E2_2^+ + E2_1^+)$ is 4 KeV only. One of the other important parameter which hints at shape of the nucleus is the energy difference $\Delta E = E3_1^+ - (2E2_1^+ + E4_1^+)$ whose value for ^{232}Th is 570, which is supposed to be zero for γ – soft nucleus. This is a significant indicator to consider this nucleus as rigid rotor. The OES effect represents the relative displacement of the odd angular momentum levels of the γ - band with respect to their neighbouring levels with even angular momentum. This is a long known effect which is clearly established in even - even nuclei [1]. As far as odd even staggering (OES) in γ – band is considered, it is supposed to show very little value for staggering indices S(I) at low spin and low energy. But, while S (I) calculated using following relation –

$$S(I) = \frac{[E(I+1) - E(I)] - [E(I) - E(I-1)]}{E2_1^+}$$

an experimental staggering indices are plotted vs. spin for ^{232}Th, it exhibits the zigzag variation from S (4) to S (8) without change in sign (all positive). The sign of staggering indices has reversed for S (9) and S (10). The nucleus is one among the five that shows the staggering associated with triaxial shapes according to McCutchen et al [2] who plotted S (I) versus angular momentum (I) exhibiting minimum value of S (I) at odd angular momentum. The general behavior of the odd – even staggering in rotational region has been studied in terms of the ground – γ band mixing interaction using Vector Boson Model (VBM) [3]. The new relation is used to study the OES in γ – band as –

$$Stg(L) = [10E(L+1) + 5E(L-1) + E(L+3)] - [10E(L) + 5E(L+2) + E(L-2)]$$

They obtained a clearly pronounced staggering pattern i.e. a zigzagging behavior of the quantity Stg (L) as a function of angular momentum (L). A regular change in signs of Stg (L) between odd and even levels with the amplitude increasing with angular momentum up to L = 12 – 13. Their method has been quoted as most effective one for the studying OES in collective γ – band (ΔL = 1).

We undertake the present work to treat ^{232}Th with a soft rotor formula used recently by C. Bihari et al [4] for γ – band and modified by J. B. Gupta et al [5]. It describes energy in terms of moment of inertia and softness parameter –

$$E(I) = E_K + \frac{I(I+1)}{2\theta_0(1+\sigma I)}$$

The energies have been calculated using two parameters i.e. moment of inertia and θ_γ and get energy constant E_K. Interestingly the value of softness parameter (σ) is found to be zero (Table – I) which is expected also for rigid rotor.

At low spin and low energy, there is an excellent matching in SRF values with experimental values. Being $\sigma = 0$, it is inferred that nucleus while rotates and generate γ – band up to spin I = 7, there is no centrifugal stretching or coriolis interaction. Thus Nucleus may be rigid. The staggering indices S (L) in experiment has been listed in table II. The alternative positive and negative values from S (8) onwards in table II indicate the γ – soft nature of nucleus and thus there is a phase transition at spin I = 8.

We have taken transition rates for the consideration and for the test of phase transition at $I = 8$ (table III). The increasing trend of experimental cascade in rotational band is reversed at $10_1^+ \rightarrow 8_1^+$. Although, more information is needed for looking at trend of transition rates in γ – band also to comment with confidence on phase transition but the values of these B (E2) rates have been found to be close to the rigid rotor estimates.

Table – I

γ – band energies in KeV calculated from SRF

Parameters: E_k= 0.74, $\boldsymbol{\sigma = 0}$, $\boldsymbol{\theta_\gamma = 66.67}$		
Energy	**Exp**	**SRF**
$\mathbf{2_1^+}$	785	785
$\mathbf{3_1^+}$	829	830
$\mathbf{4_2^+}$	890	890
$\mathbf{5_1^+}$	960	965
$\mathbf{6_2^+}$	1051	1055
$\mathbf{7_1^+}$	1142	1160
$\mathbf{8_2^+}$	1260	1230
$\mathbf{9_1^+}$	1370	1415
$\mathbf{10_2^+}$	1513	1565
$\mathbf{11_1^+}$	1640	1730

Table II

Staggering indices for ^{232}Th

S(4)	+0.33
S(5)	+0.19
S(6)	+0.42
S(7)	+0.006
S(8)	+0.53
S(9)	-0.15
S(10)	+0.65
S(11)	−0.31

Table III

B(E2, $I_f \rightarrow I_i$) as e^2b^2 for ^{232}Th

Transition	**Exp.**	**Rigid rotor**
$\mathbf{2_1^+ \rightarrow 0_1^+}$	1.850	1.800
$\mathbf{2_2^+ \rightarrow 0_1^+}$	0.024	0.049
$\mathbf{2_2^+ \rightarrow 2_1^+}$	0.055	0.088
$\mathbf{6_1^+ \rightarrow 4_1^+}$	3.110	2.980
$\mathbf{8_1^+ \rightarrow 6_1^+}$	5.450	3.160
$\mathbf{10_1^+ \rightarrow 8_1^+}$	4.990	3.280

The suggestions of ref. [6] for taking moment of inertia of γ – vibrational band as 4.36 times that of the moment of inertia of ground rotational band seems to be irrelevant. In the present work $\theta_\gamma = 66.67, \sigma = 0$ for γ – rotational band and the value of the θ_g is 60.8 for yrast band. This is significant here that θ_g and θ_γ are nearly same as the value of $\theta_\gamma = 66.67$ and both of the bands are taken as rotational. The moment of inertia for rotation to produce yrast and γ – band are nearly same which contradicts Jolos and Brentano view point [6]. At the same time both the bands are found to be rotational in the present study.

References:

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