Novel solution of power law for Gamma-bands

J.B. Gupta
Ramjas College, University of Delhi, Delhi 110007

For the ground band level energies of shape transitional nuclei, there exist several alternative formulae, with two or more terms in a series expansion, and with different degree of validity in different regions of the nuclear chart. Most of them are based on the concept of perturbation expansion. The Nuclear Softness Model (NSM) [1] or Soft Rotor formula (SRF) [2] uses the concept of the change in MoI proportional to the spin angular momentum. Long ago, Gupta *et al*. [3] proposed a single term formula, named the power index formula (1)

$$E(I) = a\, I^b. \qquad (1)$$

This single term formula is based on the generalization of the concept of weighted geometric mean of the two terms, viz. the vibration energy and the rotational energy terms. Here index 'b' is a non integer <2.0. Validity of the formula was demonstrated for the ground state bands of a vast number of A=150-200 nuclei, spherical/deformed and transitional [3]. Mittal *et al*, [4] tested the same for the N<82 nuclear region.

In its further application, [5] it was shown that the deviations from the linearity relation

$$R_{I/2} = R_{4/2}[I(I-2)/8] - [I(I-4)/4] \qquad (2)$$

which is based on the rotation vibration expression

$$E(I) = a\, I(I+1) + bI \qquad (3)$$

----------↑

Results

The level energies from the power index formula (5) agree fairly with experimental energies with relative root mean square error of less than 5 % (see last column divided by mean level energy in Table 1. This holds for on a $R_{12/2}$ versus $R_{4/2}$ plot could be well reproduced by the power index formula. The power law also covers all the five analytical symmetries of rotor Model, spherical Vibrator Model, γ-unstable rotor, analytical critical symmetries E(5) and X(5), and all the other intermediate cases [6].

In the present work we illustrate its validity and application to reproduce the rotational bands built on the γ-vibrational band head 2_γ. In the application of the SRF/NSM formula to the γ-vibrational bands [7] we used the concept of a virtual level below the band head corresponding to virtual spin zero.

$$E(I) = EK + I(I+1)/\theta_0(1+\sigma I). \qquad (4)$$

Therein we also showed that the value of EK is almost equal to the energy $[E(2_\gamma) - E(2_g)]$. A plot of the two quantities yields a diagonal. Considering it as a valid approximation, the energy of the K=2 band levels may be represented by

$$E(I) - EK = a\, I^b. \qquad (5)$$

Writing it for 2_γ and all other states of spin I and by division, one can determine both; 'b'; and 'a'. Alternatively one can take the log of both sides of Eq. (5), then 'b' is given by their division. The average values can be used to calculate the level energies.

the well deformed nuclei as well as for vibrational. This validates the application of the formula (5) for the K=2 γ-vibrational bands

.

Discussion

The level energies from power law are closer to experiment compared to SRF for the deformed and shape transitional nuclei. Odd spin energies are also well given for deformed nuclei with small odd-even staggering. A detailed Table will be presented.

A major advantage of the power index formula over other expression in use, is that it yields almost constant values of the index 'b' and the coefficient 'a' with increasing spin I. The latter corresponds to the moment of inertia parameter used in NSM/SRF or in rotation vibration formulae, which assumes unrealistic values at spin I =10 for the shape transitional nuclei. Also a better reproduction of level energies is achieved. The formula (5) is equally valid for the K=0 β-bands

Table 1. K=2 band energies (keV) from the power index formula. Upper row is for experiment, lower for the power law.

.

	$R_{4/2}$	2_g	2_γ	4_γ	6_γ	8_γ	10_γ	Rmsd
^{162}Er	3.230	102	900.7	1128.1	1459.5	1872.7	2346.5	
PL				1129.0	1455.4	1867.9	2359.1	7.0
^{156}Dy	2.933	137.3	890.5	1168.5	1525.2	1958.6	2448.0	
PL				1160.3	1524.0	1965.5	2475.8	15
^{158}Dy	3.206	98.9	946.2	1163.8	1486.4	1892.8		
PL				1167.9	1484.0	1883.3		6
^{156}Gd	3.239	88.97	1154.2	1355.4	1643.7	2011.4	2442.4	
PL				1355.2	1644.0	2010.4	2447.9	2
^{152}Sm	3.099	121.8	1085.9	1371.7	1728.2	23139.7	2662.5	
PL				1342.4	1705.2	2158.4	2693.2	44
^{102}Ru	2.329	475.1	1103.0	1799.1	2586.6	3394.9	4294.7	
PL				1771.0	2564.4	3428.9	4357.3	38
^{118}Xe	2.404	337.3	928.1	1441.2	1997.0	2624.5	3255	
PL				1408.1	1970.1	2590.3	3257.6	36

References

[1] R.K. Gupta, Phys. Lett. **B36**, 173 (1971).
[2] P von Brentano et al., Phys.Rev.C**69**, 044314 (2004).
[3] J.B. Gupta, A.K. Kavathekar and R. Sharma, Phys. Scr., **51**, 316 (1995).
[4] H.M. Mittal, V. Devi and J.B. Gupta, Phys. Scr. **81**, 2502 (2010).
[5] J.B. Gupta, Int. J. Mod. Phys **E22**(8), 1350064 (2013).
[6] J.B. Gupta, Int. J. Mod. Phys **E22** (5), 1350023 (2013).
[7] J.B. Gupta, S. Sharma, V. Katoch, Pramana J Physics, **81**, 75 (2013).