Empirical Formula for two neutrino double beta decay

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Introduction

The double beta (2β) decay is a rare nuclear weak process in which two neutrons in the nucleus are converted into two protons, and two electrons and two electron antineutrinos are emitted. The process can be thought as a sum of 2β decays. For the double beta decay to be possible, the final nucleus must have a larger binding energy than the original nucleus. More than eleven isotopes (^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}C, ^{128,130}Te, ^{136}Xe, ^{150}Nd, ^{238}U etc.) have been experimentally observed undergoing two-neutrino 2β decay [1]. The present work aims to develop an empirical formula for computing two neutrino 2β decay half-lives.

The Model

The two neutrino 2β decay rate can be expressed as a product of independent factors such as phase-space factors $G^{2\nu}$ and the nuclear matrix elements $M^{2\nu}$. The expression for computing the half life time for two neutrino 2β decay is given as,

$$T_{1/2}^{-1} = G^{2\nu} \left| M^{2\nu} \right|^2 \qquad (1)$$

Phase Space Factor

The key ingredients for the evaluation of phase space factors in single and 2β decay are the scattering wave functions. Another quantity of interest in the evaluation of phase space factor is the excitation energy of the intermediate nucleus with respect to the average of the initial and final ground state. In the present work we have computed the phase space factor in two ways.

Formula I

The Phase space factor for two neutrino 2β decay can be approximated in terms of highest powers in T in their approximate analytical expression obtained by using the Primakoff-Rosen approximation [2] for the Coulomb distortion of electron wave function at the nuclear surface. Thus the expression for phase space factor [3] is

$$G^{2\nu} = \frac{1}{7} g_0 \frac{T^{11}}{1980} \qquad (2)$$

The coupling constant g_0=3.78x10-24g_A^4yr^{-1}, g_A=1.254 and T is the maximum kinetic energy release. The expression for T is given as,

$$T = (m_A - m_B - 2m_e)/m_e \qquad (3)$$

Formula II

The phase space factor is depending on the energy decay $Q_{\beta\beta}$ and nuclear charge Z. Figure 1 represent the plot of phase space factor versus ZQ^3 for various isotopes undergoing two neutrino 2β decay. The phase space factor is taken from ref [4]. From the observed dependence of ZQ^3 and Z^{2}Q^6 of the plots we have developed a semi empirical formula for the phase space factor. Using ZQ^3, Z^{2}Q^6 and Z^{3}Q^9 as variables, a new formula is obtained by making least-squares fit to the data and is given as,

$$\log_{10}(G^{2\nu}) = a + bZQ^3 + cZ^2Q^6 + dZ^3Q^9 \qquad (4)$$

The constants are, a = -20.5256, b= 0.00488, c= -2.58601x10^{-6} and d= 5.33951x10^{-10}

Figure 2 represents the comparison of computed phase space factor by using the above two expressions with those obtained from the values of Vogel [4] for two neutrino double beta decay from various isotopes. It is found from the plot that the computed values by using formula II are in better agreement with the values of Vogel [4].

Nuclear Matrix Element

The 2β decay rate is a steep function of the energy carried by the outgoing leptons (i.e. of the decay Q-value). Hence, transitions with larger Q-value are easier to observe. Figure 3 represents the plot of nuclear matrix element values taken from [3] versus $Z^{-1/3}$(N-Z) for various isotopes undergoing two neutrino 2β decay. From the observed dependence of $Z^{-1/3}$(N-Z) and decay energy on the nuclear matrix elements, a new formula is obtained by making least-squares fit to the nuclear matrix elements data and is given as,

$$M^{2\nu} = a + b\exp\left(-18.00796[Z^{-1/3}(N-Z) - c]^2\right) + dQ^{-1/2} + eQ^{-1} + fQ^{-3/2} + gQ^{-2} \quad (5)$$

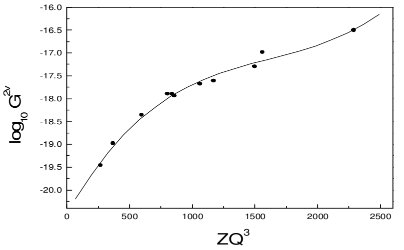

Fig. 1 The plot of phase space factor versus ZQ^3 for various isotopes undergoing 2β(2ν) decay

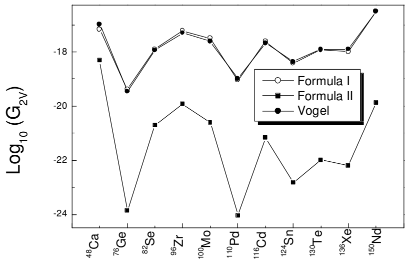

Fig. 2 The plot of comparison of phase space factor for various isotopes undergoing 2β(2ν) decay

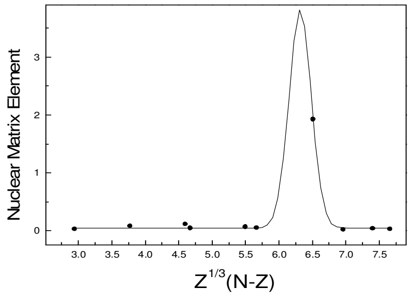

Fig. 3 The plot of nuclear matrix element versus $Z^{1/3}$(N-Z) for various isotopes undergoing 2β(2ν) decay

The constants are, a = 116.06437, b=3.782184, c=6.31711, d= -810.5068, e= 2108.95565, f= -2422.96473 and g= 1037.30225.

The computed nuclear matrix element using the present formula and those from Ref [3] are shown in the Table 1. Figure 4 represents the comparison of computed half lives with the experimental values. It found from the plot that the computed values by using formula II are in good agreement with the experimental half lives.

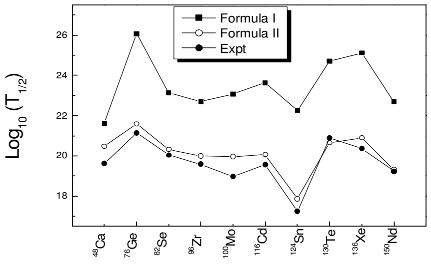

Fig. 4 The plot of comparison of half life time with the present and with experimental values for various isotopes undergoing 2β(2ν) decay

Table 1: The comparison of the computed nuclear matrix element using the present formula and with the values of ref [3]

Isotope	Q value	$M_{2\nu}$	
	(KeV)	Present	Ref [3]
^{48}Ca	4273.7	0.022338	0.024
^{76}Ge	2039.1	0.078271	0.074
^{82}Se	2995.5	0.061394	0.046
^{96}Zr	3347.7	0.040666	0.038
^{100}Mo	3035.0	0.058292	0.106
^{116}Cd	2809.1	0.059320	0.059
^{124}Sn	2287.7	1.897271	<1.920
^{130}Te	2530.3	0.043281	<0.016
^{136}Xe	2461.9	0.034912	<0.030
^{150}Nd	3367.3	0.039274	0.022

References

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