A simple approach of extracting β from $E2_1^+$

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It has been observed that the values of β calculated from energy and transition rate are not closer [1, 2]. The reason for this gap in the values of β was assigned to employ the β – moment of inertia relation of hydrodynamic model. In the present work, a simple approach is proposed to evaluate β from $E2_1^+$ using the semi empirical relation of Grodzins [3] which connects $E2_1^+$ with $B(E2; 2_1^+ \to 0_1^+)$.

$$E2_1^+ . B(E2; 2_1^+ \to 0_1^+) = (2.5 \pm 1) \times 10^{-3} Z^2 A^{-1} \text{ (MeV.e}^2\text{b}^2) \quad (1)$$

For symmetric nucleus it has been modified as –

$$E2_1^+ . B(E2; 2_1^+ \to 0_1^+) = 2.5 \times 10^{-3} Z^2 A^{-1} \text{ (MeV.e}^2\text{b}^2) \quad (2)$$

Using equation (2), the moment of inertia parameter J_0 can be related to an effective value of β [4] for symmetric nucleus-

$$\frac{6\hbar^2}{2J_0} = \frac{1224}{\beta^2 A^{7/3}} \quad (3)$$

The hydrodynamic relation [4] relates $E2_1^+$ with moment of inertia J_0 and asymmetric parameter γ for asymmetric nucleus (γ ≠ 0) as

$$E2_1^+ = \frac{6\hbar^2}{2J_0} \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma} \quad (4)$$

For symmetric nucleus (γ = 0), equation (4) reduces to

$$E2_1^+ = \frac{6\hbar^2}{2J_0} \quad (5)$$

Equation (3) and (5) gives a relation between $E2_1^+$ and β for symmetric nucleus as –

$$E2_1^+ = \frac{1224}{\beta^2 A^{7/3}} \quad (6)$$

From equation (3) and (4), we can get

$$E2_1^+ = \frac{1224}{\beta^2 A^{7/3}} \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma} \quad (7)$$

Equation (7) allows us to extract β from $E2_1^+$ for asymmetric nucleus $A.E2_1^+.B(E2; 2_1^+ \to 0_1^+)/Z^2$ values are evaluated for isotopic chains of Xe, Ba, Ce and Hf and are written under C in the table – I. For most of the nuclei the value of C is more than 2.5. Equation (6) should become valid for asymmetric nuclei, if the 2.5 of the Grodzins product is replaced by the exact value C. Thus, equation (6) may be rewritten for asymmetric nucleus as –

$$E2_1^+ = \frac{1224 \times (C/2.5)}{\beta^2 A^{7/3}} \quad (8)$$

On comparing equation (7) and (8) we find that

$$\frac{C}{2.5} = \frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma}$$

This is justified for symmetric nucleus. Also since for C = 2.5, the value of asymmetric parameter γ is zero. We can assume safely that $C/2.5$ takes care of the factor $\frac{9-\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma}$. Thus, the controversy of the factor used in earlier work that produces unwanted values of β has been avoided.

β Values are evaluated from equation (8) β_e and β_b values have been evaluated from early work [1, 2]. Following observations are made

1. Present values of β are closer to the β_b.
2. Maximum variation in the values of β from β_b is 26.8 as noted by previous workers for ^{134}Ce which is reduced to 4.8 only in present work.

The above procedure has been found useful earlier also while extracting β from $E2_1^+$ for Mo, Ru and Pd isotopic chain [5].

Table - I

Data of column 2,3, 5 and 6 are taken from reference [1, 2]. Grodzins constant C, presently calculated β_e and variation in percent are listed in column 4, 7, 8 and 9.

Nucl.	$E2_1^+$ (MeV)	$B(E2;2_1^+ \rightarrow 0_1^+)$ (e^2b^2)	C	β_b	β_e	β_e (Present work)	Percentage variation in β_e from β_b with	
							Present values	Old values
^{122}Xe	0.331	0.265(20)	3.670	0.261(20)	0.26	0.277	6.1	-0.4
^{124}Xe	0.354	0.240(20)	3.610	0.250(19)	0.25	0.260	4.0	0
^{126}Xe	0.389	0.154(5)	2.588	0.191(6)	0.24	0.203	6.3	25.6
^{128}Xe	0.443	0.150(8)	2.917	0.186(9)	0.22	0.199	7.0	18.3
^{130}Xe	0.536	0.130(10)	3.106	0.170(13)	0.20	0.183	5.9	17.6
^{132}Xe	0.668	0.092(6)	2.780	0.141(9)	0.18	0.152	7.8	27.6
^{124}Ba	0.230	0.401(9)	3.647	0.295(6)	0.30	0.329	11.5	1.7
^{126}Ba	0.256	0.380(4)	3.910	0.284(3)	0.28	0.316	11.2	-0.1
^{128}Ba	0.284	0.276(17)	3.200	0.240(14)	0.26	0.266	10.8	8.3
^{130}Ba	0.357	$\leq$0.23	3.400	$\leq$0.217	0.24	0.240	10.6	10.3
^{132}Ba	0.465	0.172(12)	3.370	0.190(13)	0.21	0.203	6.8	10.5
^{134}Ba	0.604	0.136(32)	3.510	0.164(38)	0.18	0.177	7.9	9.7
^{128}Ce	0.207	0.430(36)	3.387	0.289(24)	0.26	0.310	7.2	-10.0
^{130}Ce	0.254	0.346(18)	3.396	0.274(14)	0.30	0.285	4.0	9.5
^{132}Ce	0.325	0.354(28)	4.445	0.269(21)	0.28	0.282	4.8	4.0
^{134}Ce	0.409	0.206(18)	3.356	0.205(17)	0.26	0.215	4.8	26.8
^{136}Ce	0.552	-	3.000	-	0.22	0.168	-	-
^{164}Hf	211.1	-	3.50	-	0.22	0.238	-	-
^{166}Hf	158.5	0.692(36)	3.51	0.255(8)	0.25	0.284	11.3	6.0
^{168}Hf	124.0	0.858(40)	3.45	0.284(8)	0.27	0.30	5.6	-4.9
^{170}Hf	100.8	1.00(2)	3.28	0.300(2)	0.29	0.30	0.3	-3.3
^{172}Hf	95.2	0.878(60)	2.77	0.283(11)	0.29	0.297	4.9	2.4
^{174}Hf	91.0	0.960(58)	2.93	0.290(11)	0.30	0.316	8.9	3.3
^{176}Hf	88.4	1.054(20)	3.16	0.303(15)	0.29	0.32	5.6	4.2
^{178}Hf	93.2	0.964(12)	3.01	0.28	0.28	0.30	7.1	0.0
^{180}Hf	93.3	0.930(16)	3.00	0.276(2)	0.28	0.30	8.7	1.4
^{182}Hf	97.8	-	3.00	-	0.27	0.286	-	-

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