Two Body Perturbing Resonances in a Three Body System: Origin of σ-Meson in N-N Interaction

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There are many missing links in the nuclear structure many body problems, such as the treatment of the repulsive core part of the N-N interaction, inclusion of non-perturbative effects in the many body solution, proper inclusion of tensor interaction, in the field theoretic formulation[1] the necessity yet still unobserved σ-meson and many other such problems. One of the most intrigueing problems is to explain the elusiveness of the σ-meson, supposed to be a two-pion resonance which has not been seen in the π - π scattering experiments. The σ-meson is a requirement even for the explanation of the N-N scattering data. It is usually argued that it may be too broad a resonance or that it may be manifestation of an approximation to the dominant contribution in the two pion exchanges. If its mass together with the coupling constants for all the other mesons are fitted to the experimental N-N scattering data then they can be reproduced well in the OBEP potential.

Durso *et al* [2, 3] have shown that the correlated π π S-wave contribution can be well approximated by the exchange of a scalar-isoscalar boson with broad mass distribution and this correlated 2π-exchange contribution provides about $\frac{2}{3}$ of the total 2π-exchange. This correlated 2π-exchange (denoted by σ') contribution with $\frac{g^2_{\sigma'}}{4\pi} = 10$, $m_{\sigma'} = 662.5$ MeV and $\Gamma_{\sigma'} = 524.5$ MeV provides a realistic description of the long- and intermediate-range part of the N-N interaction. This clearly indicates that the width of this σ-meson is very large, in fact it is comparable to its mass. The problem now is how to obtain this σ'-meson.

We see that all the efforts using the 2-body model have not yielded desired results. Therefore we propose to approach this problem from a different 3-body problem angle. In this 3-body approach we take the time uncertainty such that the nucleon-N goes to, N$\rightarrow \pi + \Delta$ where the Δ further goes to, $\Delta \rightarrow \pi + N$. Therefore effectively the time-energy uncertainty allows us to have N$\rightarrow \pi + N + \pi$ where both the $\underline{\pi + N}$ arms of this 3-body system correspond to a Δ-resonance. We know that for a 3-particle system the Hamiltonian H is written as:

$$H = T_{12} + T_{13} + V_{12} \quad + V_{13} + V_{23} + \frac{\hbar^2}{m_1}\vec{\nabla}_{12} \cdot \vec{\nabla}_{13} \qquad (1)$$

Here the coupling term $\frac{\hbar^2}{m_1}\vec{\nabla}_{12} \cdot \vec{\nabla}_{13}$ couples the 1-2 motion with that of the 1-3 motion. Here the kinetic energy operator $T_{ij} = \frac{\hbar^2}{2\mu_{ij}}\nabla^2_{ij}$ with $\mu_{ij} = \frac{m_i m_j}{m_i + m_j}$ being the reduced mass.

Similar expression is obtained in the classical case of the 3-body problem where the $-i\hbar\vec{\nabla}_{ij}$ of the quantum case is replaced by the corresponding momentum vector $\vec{p}_{ij}$. Now if there are resonance conditions satisfied for 1-2 and 1-3 motions separately then we have for harmonic oscillator case for example:

$$H_{ij} = -\frac{\hbar^2}{2\mu_{ij}}\nabla^2_{ij} + \frac{1}{2}\mu_{ij}\omega^2 r^2_{ij} \qquad (2)$$

The solution of the corresponding Schrodinger equation is:

$$\Phi_{ij}(\vec{r}_{ij}) = \varphi_{nl}(r_{ij}) Y_{lm}(\hat{r}_{ij}) \qquad (3)$$

In order to evaluate the coupling term $\frac{\hbar^2}{m_1}\vec{\nabla}_{12} \cdot \vec{\nabla}_{13}$ for these bound wave functions one can start by assuming that the motions 1-2 and 1-3 are restricted to one plane, say x-z plane. This will have ϕ angle wave function to unity and we will have:

$$\Phi_{ij}(\vec{r}_{ij}) = \varphi_{nl}(r_{ij}) Y_{l0}(\hat{r}_{ij}) \qquad (4)$$

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or in other words

$$\Phi_{ij}(\vec{r}_{ij}) = \varphi_{nl}(r_{ij})P_l(cos\theta_{ij}). \quad (5)$$

Where the $P_l(cos\theta)$ is the normal Legendre polynomial. Now if the radial motion is in its lowest state then the radial quantum number, $n = 0$, and we have:

$$\Phi_{ij}(\vec{r}_{ij}) = \varphi_{0l}(r_{ij})P_l(cos\theta_{ij}), \quad (6)$$

where

$$\varphi_{0l}(r) = A_l r^{l+1} e^{-\nu r^2} \quad (7)$$

A_l being the normalization constant. Now we can evaluate the expression for $\vec{\nabla}\Phi(\vec{r})$ required for the evaluation of the three-body coupling term $\frac{\hbar^2}{m_1}\vec{\nabla}\Phi(\vec{r}_{12}) \cdot \vec{\nabla}\Phi(\vec{r}_{13})$. We get

$$\begin{aligned}\vec{\nabla}\Phi(\vec{r}) &= A_l r^l e^{-\nu r^2}[P_l(cos\theta)\{(l+1) - 2\nu r^2\}\hat{r} \\ &+ l\hat{\theta}\{cot(\theta)P_l(cos(\theta) \\ &- cosec(\theta)P_{l-1}(cos(\theta))\}] \quad (8)\end{aligned}$$

We now have all the required expressions required to get the expectation value of the coupling term $\frac{\hbar^2}{m_1}\vec{\nabla}\Phi(\vec{r}_{12}) \cdot \vec{\nabla}\Phi(\vec{r}_{13})$. We now get it as a three 4-dimensional integral over r_{12}, r_{13}, θ_{12} and θ_{13} which can be evaluated fairly easily using Gaussian quadrature method. It is further to be stressed that though the expression looks complicated but is easy to understand in terms of scalar product of two momentum vectors.

$$\begin{aligned}&\left\langle \Phi(\vec{r}_{12})\Phi(\vec{r}_{13})|\vec{\nabla}_{12} \cdot \vec{\nabla}_{13}|\Phi(\vec{r}_{12})\Phi(\vec{r}_{13}) \right\rangle \\ &= A_l^4 \int \varphi_{0l}^2(r_{12})P_l(cos\theta_{12}) \\ &\varphi_{0l}^2(r_{13})P_l(cos\theta_{13}) \\ &[P_l(cos\theta_{12})\{(l+1) - 2\nu r_{12}^2\}\hat{r}_{12} \\ &+ l\hat{\theta}_{12}\{cot\theta_{12}P_l(cos\theta_{12}) \\ &- cosec\theta_{12}P_{l-1}(cos\theta_{12})\}] \\ &\cdot [P_l(cos\theta_{13})\{(l+1) - 2\nu r_{13}^2\}\hat{r}_{13} \\ &+ l\hat{\theta}_{13}\{cot\theta_{13}P_l(cos\theta_{13}) \\ &- cosec\theta_{13}P_{l-1}(cos\theta_{13})\}] \\ &r_{12}dr_{12}d\Omega_{12}r_{13}dr_{13}d\Omega_{13}\end{aligned}$$

One sees that this perturbation is going to repeat over and over again until at least one of the arms gets rid off the resonance or the bound state to the one that is out of phase with the other arm and thus producing a coupling of the two π-mesons to some thing similar to σ-meson. Thus the two Δ's in the two arms which correspond to an energy of 600 MeV above the N-state will be split into a 4-state system with one Δ and two -Δ's being driven off resonance by the perturbation term each one below or one above the resonance energy of 300 MeV by a shift of $\sim$100 MeV. This resonance kill mechanism changes these to, for example $\sim$200 MeV and thus effectively producing a sigma meson resonance of $\sim$300 MeV (being the Δ resonance excitation energy) times 2 minus 100 (the shift in resonance energy) times 2 approximately equal to $\sim$400 MeV sigma meson mass. Such a sigma mass is close to that commonly used in Relativistic Mean Field (RMF)[1] theory of nuclei. It could be that only one Δ resonance resonance is killed then the corresponding sigma mass is close to $\sim$500 MeV.On the other hand if the shift is toward the increasing side of the resonance mass then there could be another two sigma mesons of 700 and 800 MeV mass. This σ-mass also will depend upon the coupling of the two P^{33} resonances(Δ)'s. If they couple to S-state then the σ-meson will have a different mass as compared to the one where they couple to D-state.

A similar mechanism has been operating between the missing Saturn's rings and its moons, this 3-body perturbation removes the particles from the Saturn's disc whenever their frequency ratios are rational numbers. Thus correlating the motion of that disc particle and that of the moon. Similar phenomena explained the missing Asteroids in between the Jupiter and Mars known as Kirkwood gaps.

References

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