Near barrier fusion cross sections for medium-heavy nuclei

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Introduction

Fusion reactions are widely used in nuclear physics to produce nuclei far from β-stability line and superheavy nuclei. Burning of stars is also associated with reactions involving sub-barrier fusion of nuclei. Fusion occurs when interacting nuclei overcome the barrier formed by the sum of nuclear, Coulomb and centrifugal potentials. If kinetic energy in CM is below barrier height, fusion occurs through quantum tunneling. Aim of the present work is to obtain nuclear fusion cross sections involving medium and heavy nucleus-nucleus systems.

Fusion barrier distribution & cross section

In order to reproduce shapes of experimentally observed fusion excitation functions, particularly at low, near-threshold energies, it is necessary to assume a distribution of the fusion barrier heights [1], the effect that results from the coupling to other than relative distance degrees of freedom. A Gaussian shape for the distribution $D(B)$ of the fusion barrier heights is assumed which is given by

$$D(B) = \frac{1}{\sqrt{2\pi}\sigma_B} \exp\Big[-\frac{(B-B_0)^2}{2\sigma_B^2}\Big] \quad (1)$$

where the two parameters, the mean barrier height B_0 and the distribution width σ_B, to be determined individually for each reaction.

The energy dependence of the fusion cross section is obtained by folding [2] the barrier distribution provided by Eq.(1), with classical expression for the fusion cross section given by

$$\sigma_f(B) = \pi R_B^2 \Big[1 - \frac{B}{E}\Big] \quad \text{for} \quad B \le E$$
$$= 0 \quad \text{for} \quad B \ge E \quad (2)$$

where the effective barrier radius R_B denotes relative distance corresponding to the position of the barrier approximately, which yields

$$\sigma_c(E) = \int_0^\infty \sigma_f(B) D(B) dB \quad (3)$$
$$= \pi R_B^2 \frac{\sigma_B}{E\sqrt{2\pi}} \Big[\xi\sqrt{\pi}\Big\{\text{erf}\xi + \text{erf}\xi_0\Big\} + e^{-\xi^2} + e^{-\xi_0^2}\Big]$$

$$\xi = (E - B_0)/(\sigma_B\sqrt{2}), \; \xi_0 = B_0/(\sigma_B\sqrt{2}) \quad (4)$$

and erfξ is the Gaussian error integral of argument ξ. The parameters B_0, σ_B along with R_B is determined by fitting Eq.(3) along with Eq.(4) to a given fusion excitation function.

The task of estimating σ_c rests on predicting B_0, σ_B, R_B values for a particular reaction. Since B_0 is essentially the mean height of Coulomb barrier, it must be a function of Coulomb parameter $z = Z_1 Z_2/(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}})$ in the vicinity of the barrier. To a crudest approximation it is just ze^2/r_{0c} where e and r_{0c} are the elementary charge and Coulomb radius parameter, respectively. Thus, a fair extrapolation formula for B_0 can be expressed by

$$B_0 = a_1 z + a_2 z^2 + a_3 z^3 \quad (5)$$

where the coefficients a_1, a_2 and a_3 are to be fixed from fitting the data for a large number a reacting pair of nuclei and expecting a_1 to be $\sim e^2/r_{0c}$ while a_2 and a_3 to be orders of magnitude smaller. The extrapolation of the trend of σ_B is more difficult which arises out of nuclear deformation and vibration and quantal barrier penetrability. Assuming all possible orientations of a nucleus i with static deformation $\beta_2(i)$, one obtains the variation of the effective radius R_i with standard deviation [3]

$$\omega_i = \frac{\beta_2(i) R_i}{\sqrt{4\pi}}. \quad (6)$$

Multipolarities higher than quadrupole are disregarded. Thus distribution of the resulting surface-surface distance, for fixed distance between centers of mass of two nuclei, leads to standard deviation σ_i of barrier distribution:

$$\sigma_i = \omega_i \Big|\frac{\partial V}{\partial r}\Big|_{r=R} \approx \frac{CB_0}{R}\frac{\beta_2(i) R_i}{\sqrt{4\pi}} \quad (7)$$

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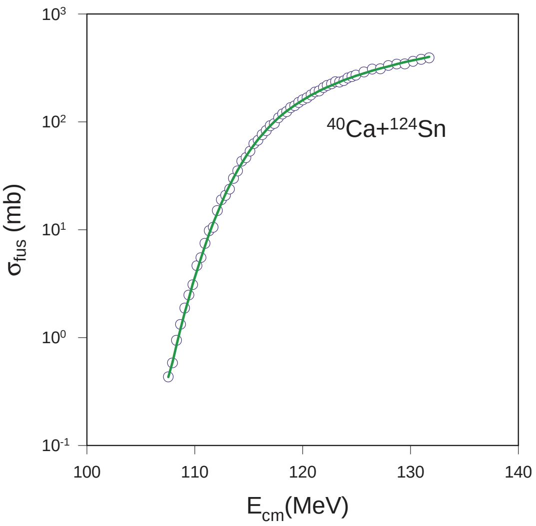

FIG. 1: Comparison of the measured fusion excitation functions (full circles) for ^{40}Ca+^{124}Sn with predictions (solid line) of diffused barrier formula.

where C is a dimensionless adjustable parameter whose value is expected to be of the order of unity. Now the σ_B can be given by

$$\sigma_B = \sqrt{\Sigma_{i=0}^2 \sigma_i^2} = \frac{CB_0}{R_1}\sqrt{\omega_1^2 + \omega_2^2 + \omega_0^2} \quad (8)$$

where R=R_1+R_2=$r_0[A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}}]$, ω_1, ω_2 are the standard deviations of the radius vectors specifying the surfaces of the projectile and target, whose mean radii are R_1, R_2 and whose quadrupole deformation parameters are $\beta_2(1)$, $\beta_2(2)$, respectively. The quantity ω_0 in Eq.(8) is an adjustable parameter that accounts for nuclear vibrations and quantal barrier penetrability. The third quantity R_B should have a form of $r_{0B}(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$, where r_{0B} can be fixed from fitting data for a large number of nuclei.

Calculations and Results

The values of B_0, σ_B and R_B are obtained by least-square fitting the experimental data [4]. These are listed in Table-I. The nonlinear least square fits with B_0, σ_B and R_B values for a set of fiftyone pair of nuclei yield from Eq.[5], a_1=0.912025 MeV, a_2=0.600849 × 10^{-4} MeV and a_3=0.315525×10^{-5} MeV and from Eq.[8], C=0.34, r_0=1.16 fm and ω_0=0.59 fm whereas r_B=1.126 fm. Predictions of the present calculations for 250 MeV ^{48}Ca incident beam on few target nuclei that are especially important for planning experiments for synthesizing new superheavy elements are provided in Table-II.

TABLE I: Extracted values of B_0, σ_B and R_B, deduced from measured fusion excitation functions.

Reaction	z	B_0 [MeV]	σ_B [MeV]	R_B [fm]
^{16}O+^{154}Sm	62.94	58.80	2.43	10.04
^{17}O+^{144}Sm	63.49	60.28	1.75	10.46
^{16}O+^{148}Sm	63.51	59.88	2.31	10.61
^{16}O+^{144}Sm	63.91	60.65	1.76	10.46
^{36}S+^{110}Pd	90.94	85.51	1.92	8.20
^{32}S+^{110}Pd	92.39	86.04	3.10	8.45
^{48}Ca+^{96}Zr	97.41	93.76	2.75	10.07
^{48}Ca+^{90}Zr	98.58	94.94	2.11	10.01
^{40}Ca+^{96}Zr	100.01	94.30	3.09	9.71
^{40}Ca+^{90}Zr	101.25	96.26	1.67	10.07
^{48}Ca+^{124}Sn	116.00	111.93	1.28	8.24
^{40}Ca+^{124}Sn	118.95	113.36	2.70	9.57

TABLE II: Theoretical values of B_0, σ_B, R_B and σ_c for 250 MeV ^{48}Ca beam on target nuclei.

Target Nuclei	B_0 [MeV]	σ_B [MeV]	R_B [fm]	σ_c [mb]
^{244}Pu	197.38	4.37	11.13	214.67
^{243}Am	200.19	4.44	11.12	160.34
^{245}Cm	202.37	4.56	11.14	127.18
^{248}Cm	201.72	4.55	11.17	146.21
^{249}Bk	204.12	4.60	11.18	107.47
^{249}Cf	206.74	4.66	11.18	67.87
^{252}Es	208.72	4.71	11.20	48.63
^{254}Es	208.28	4.61	11.22	56.48

Summary and Conclusion

Rigorous analysis for near-barrier fusion excitation functions is performed by using a simple diffused barrier formula derived assuming Gaussian barrier height distribution. The parameters of the distribution are extracted by fitting the predicted fusion excitation function with experimental data. Theoretical model for estimating parameters of barrier distribution is described and their values estimated. These can be used for predicting fusion cross sections for planning superheavy element syntheses.

References

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