# Masses and Decay properties of excited charmonia states

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## Introduction

Charmonium has proved a remarkable laboratory for the study of quantum chromodynamics(QCD). The study of its spectrum provides fundamental informations about the interquark potential. Since the hadron spectrum cannot be obtained directly from QCD, one has to use other methods like potential model calculations, lattice gauge theory, effective field theory, etc. to investigate hadron spectrum and its decays. Phenomenological potential models are still one of the important tools to study the hadron spectrum and its decays. These models are either relativistic or non-relativistic. The non-relativistic quark model is commonly employed for knowing the behavior of heavy hadron as this approximation provides a good description of static properties of heavy mesons such as mass spectra, while for dynamical properties such as decay, the relativistic corrections are considered.

## Formalism

For the description of the quarkonium bound states, we adopt the phenomenological potential of the form given by

$$V(r) = \frac{-4\alpha_s}{3r} + \frac{Ar^2}{(1+4Br)^{\frac{1}{2}}} - V_s \qquad (1)$$

Here, A=0.374Gev$^3$, B=1.0GeV and $V_s$ is a state dependant constant potential. Similar type of potential is used by [1] for the study of light flavor hadrons using Bethe-Salpeter Approach. The mass spectra of nS and nP waves are obtained by numerically solving the Schrödinger equation using the Mathematica notebook of the Runge-Kutta method [2]. Different degenerate n$^{2S+1}L_J$ low-lying states of $c\bar{c}$ mesonic states(nS and nP) are calculated by considering the spin-dependent part of the usual one gluon exchange potential [5] perturbatively. The quark mass $m_c = 1.28\ GeV$, while the state dependant constant potential ($V_s$ ) is given by the recursion relation,

$$V_s(n+1,l) = V_s(n) + 0.02l(3l+5) + \frac{1}{2} \quad (2)$$

The value of $V_s(n=0, l=0)$ is fixed as 0.12 $GeV$.

Apart from the masses of the low lying mesonic states, the correct predictions of the decay rates are important features of any successful model. The radiative decays of the bound $c\bar{c}$ states provide an excellent laboratory for studying charmonium decay dynamics and the light hadron spectroscopy. An electromagnetic decay occurs when the $c\bar{c}$ pair annihilates into one or more photons, which can subsequently give origin to a pair of leptons as the final state. A decay to a pair of leptons is only allowed to the states with the same quantum numbers as the photon, that is $J^{PC} = 1^{--}$; using the Van Royen-Weisskopf formula the leptonic decay width with radiative correction for the vector mesons reads:

$$\Gamma(n^3S_1 \longrightarrow l^+l^-) = \frac{4N_c\alpha^2 {e_Q}^2|R_0(0)|^2}{{M_V}^2} \times \left[1 - \frac{16}{3}\left(\frac{\alpha_S}{\pi}\right)\right] \qquad (3)$$

We have also computed the di-gamma decay

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TABLE I: Charmonium mass spectra for nL($L = 1, 2$) states (in GeV)

| State | Present | [4](in MeV) | [5] | [8] | [9] |
|---|---|---|---|---|---|
| $1^1S_0$ | 2.979 | $2983.6 \pm 0.07$ | 2.982 | $3.010 \pm 0.001$ | 2.980 |
| $1^3S_1$ | 3.096 | $3096.91 \pm 0.011$ | 3.090 | $3.085 \pm 0.001$ | 3.097 |
| $1^3P_0$ | 3.488 | $3414.75 \pm 0.31$ | 3.424 | $3.408 \pm 0.002$ | 3.392 |
| $1^3P_1$ | 3.514 | $3510.66 \pm 0.007$ | 3.505 | $3.472 \pm 0.009$ | 3.490 |
| $1^1P_1$ | 3.539 | $3525.39 \pm 0.11$ | 3.516 | $3.474 \pm 0.010$ | 3.523 |
| $1^3P_2$ | 3.565 | $3556.20 \pm 0.09$ | 3.556 | $3.503 \pm 0.024$ | 3.570 |
| $2^1S_0$ | 3.600 | $3639.4 \pm 1.3$ | 3.630 | $3.770 \pm 0.040$ | 3.631 |
| $2^3S_1$ | 3.680 | $3686.10^{+0.012}_{-0.014}$ | 3.672 | $3.739 \pm 0.046$ | 3.687 |
| $2^3P_0$ | 3.947 | - | 3.852 | $4.008 \pm 0.122$ | 3.844 |
| $2^3P_1$ | 3.972 | - | 3.925 | $4.067 \pm 0.105$ | 3.902 |
| $2^1P_1$ | 3.996 | - | 3.954 | $4.053 \pm 0.095$ | 3.920 |
| $2^3P_2$ | 4.021 | $3927.2 \pm 2.6$ | 3.972 | $4.030 \pm 0.180$ | 3.949 |
| $3^1S_0$ | 4.011 | - | 4.063 | - | 3.992 |
| $3^3S_1$ | 4.077 | $4039 \pm 1$ | 4.072 | - | 4.030 |
| $4^1S_0$ | 4.397 | - | 4.384 | - | 4.244 |
| $4^3S_1$ | 4.454 | $4421 \pm 4$ | 4.406 | - | 4.273 |

TABLE II: Di-gamma decay widths of $nS$ and $nP$ waves (in keV)

| State | Present | | [4] | [9] | [10] | [11] | [12] |
|---|---|---|---|---|---|---|---|
| | $\Gamma$ | $\Gamma_{cor}$ | | | | | |
| $\eta_c$ | 12.18 | 8.32 | $5.055 \pm 0.411$ | 10.373 | 8.5 | 7.5-10 | 10.94 |
| $\eta_c'$ | 9.21 | 6.21 | $2.147 \pm 1.580$ | 3.349 | 2.4 | 3.5-4.5 | – |
| $\chi_{c_0}$ | 9.96 | 5.06 | $2.341 \pm 0.189$ | – | 2.5 | 5.0 | 6.38 |
| $\chi_{c_2}$ | 1.35 | 0.68 | $0.528 \pm 0.404$ | – | 0.31 | 0.70 | 0.57 |
| $\chi_{c_0}'$ | 5.46 | 2.97 | – | – | – | – | – |
| $\chi_{c_2}'$ | 1.48 | 0.74 | – | – | – | – | – |

rates of the charmonia system using the expression given in [6, 7]. The spectroscopic parameters such as the masses and the resultant radial wave functions obtained from the numerical solution are used to compute the decay rates of charmonium states. In many cases of potential model predictions, the radial wave function at the origin are overestimated as far as the decay rates are concerned. In such cases, it is argued that the decay of $Q\bar{Q}$ occurs not at zero separation but at some finite radial separation ($r_c$).

TABLE III: Di-leptonic decay widths of charmonium (in keV)

| State | Present | | Exp | [9] | [13] |
|---|---|---|---|---|---|
| | $\Gamma_{r=0}$ | $\Gamma_{r=r_c}$ | | | |
| $J/\psi(1^3S_1)$ | 4.70 | 4.68 | $5.55 \pm 0.14$ | 4.94 | 1.89 |
| $\psi(2S)(2^3S_1)$ | 3.43 | 1.74 | $2.36 \pm 0.04$ | 1.67 | 1.07 |
| $\psi(3S)(3^3S_1)$ | 2.90 | 0.38 | $0.86 \pm 0.007$ | 0.95 | 0.77 |

## Results and discussions

The computed mass spectra are listed in Table I and we found an agreement with the lattice prediction as well as known experimental results [4, 8]. Di-gamma decay width obtained here are in good agreement with the experimental results and with the predictions from other theoretical models as shown in the Table II. The di-leptonic decay width computed at finite separation are found to be in good agreement with the experimental results listed in Table III.

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