# Gravitational form factors in QED and Yukawa model

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## Introduction

One of most outstanding problem in Quantum Chromodynamics (QCD) is to study the internal structure of hadrons i.e. to determine the spectrum and structure of hadrons in terms of their quark and gluon degrees of freedom[1]. The light-cone Fock state wavefunction (LCWF) $\psi_{n/H}(x_i, \vec{k_{\perp i}}, \lambda_i)$[2] has number of remarkable features. The set of LCWFs provide a frame-independent, quantum-mechanical description of hadrons at the amplitude level which are capable of encoding multi-quark and gluon momentum, helicity and flavor correlations in the form of universal process independent hadron wavefunctions. One can also construct the invariant mass operator $H_{LC} = P^+P^- - P_\perp^2$ and light-cone time operator $P^- = P^0 - P^z$ in the light-cone gauge from the QCD Lagrangian[3]. The coordinates of the light-cone Fock wavefunctions $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$ are the light-cone momentum fractions $x_i = k_i^+/P^+$, $\vec{k}_{\perp i}$ represent the relative momentum coordinates of the QCD constituents and $\lambda_i$ label the spin projections of the quarks in the $z$-direction. In the present work, we study the spin-flip matter form factor $B(q^2)$ of energy-momentum tensor for a spin-$\frac{1}{2}$ composite system. The spin-flip matter form factor receives contribution from fermion and boson constituents. This was proved that anomalous gravitomagnetic moment coupling $B(0)$ to gravity vanish for any composite system. Classically, this result was derived from the equivalence principle and from the conservation of the energy-momentum tensor. Further, it has also been shown that differentiating the wavefunction w.r.t bound state mass $M^2$ improves the behavior of wavefunctions near the end points of $x$ . In this context, we intend to check the consistency of results i.e. whether the contribution of fermion and boson constituents to $B(q^2)$, in the case of simulated model, vanishes at zero momentum transfer or not. We calculate the fermion and boson contributions to the spin-flip matter form factor in QED and Yukawa theory in simulated model. For $q^2 \neq 0$, $B(q^2)$ does not vanish. However, this model will provide a check whether the contributions to the spin-flip matter form factor vanishes at $q^2 = 0$ or not due to the Lorentz boost properties.

## Electromagnetic and Gravitational form factor

The gravitomagnetic form factors of the energy momentum tensor $A(q^2)$ and $B(q^2)$ for a spin-$\frac{1}{2}$ composite are defined as

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \bar{u}(P')\Big[A(q^2)\gamma^{(\mu}\bar{P}^{\nu)} + B(q^2)\frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P), \quad (1)$$

where $\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $q^\mu = (P' - P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$, and $u(P)$ is the spinor. The fermion and boson contributions to the spin-flip matter form factor can now be expressed in terms of the two-particle wavefunctions giving

$$B_{\rm f}(q^2) = \frac{-2M}{(q^1 - {\rm i}q^2)}\int\frac{{\rm d}^2\vec{k}_\perp {\rm d}x}{16\pi^3}\, x\Big[\psi^{\uparrow\,*}_{+\frac{1}{2}\,-1}(\vec{k}'_\perp)\psi^{\downarrow}_{+\frac{1}{2}\,-1}(\vec{k}_\perp) + \psi^{\uparrow\,*}_{-\frac{1}{2}\,+1}(\vec{k}'_\perp)\psi^{\downarrow}_{-\frac{1}{2}\,+1}(\vec{k}_\perp)\Big], \quad (2)$$

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$$B_{\rm b}(q^2) = \frac{-2M}{(q^1 - {\rm i}q^2)} \int \frac{{\rm d}^2\vec{k}_\perp {\rm d}x}{16\pi^3} (1-x) \Big[\psi^{\uparrow\,*}_{+\frac{1}{2}\,-1}(\vec{k}''_\perp)\psi^{\downarrow}_{+\frac{1}{2}\,-1}(\vec{k}_\perp) + \psi^{\uparrow\,*}_{-\frac{1}{2}\,+1}(\vec{k}''_\perp)\psi^{\downarrow}_{-\frac{1}{2}\,+1}(\vec{k}_\perp)\Big], \quad (3)$$

where

$$\vec{k}'_\perp = \vec{k}_\perp + (1-x)\vec{q}_\perp, \quad (4)$$

and

$$\vec{k}''_\perp = \vec{k}_\perp - x\vec{q}_\perp, \quad (5)$$

$$B_f(q^2) = -4M \int dx x^5 (1-x)^3 \left(M - \frac{m}{x}\right) I_1, \quad (6)$$

$$B_b(q^2) = 4M \int dx x^5 (1-x)^3 \left(M - \frac{m}{x}\right) I_2. \quad (7)$$

In order to check the behaviour of the fermion and boson constituent at zero momentum transfer we take $q_\perp^2 \to 0$ in the quantities which are dependent of $q_\perp^2$. It is found that at zero momentum transfer

$$B(0) = B_f(0) + B_b(0) = 0. \quad (8)$$

This result is in agreement with conservation of the energy momentum transfer and equivalence principle based classical arguments. For numerical calculations we have taken $M = m = 0.51$ MeV[2]. The helicity-flip boson form factor $B_b(q^2)$ of the graviton coupling to the boson constituent of the electron at one-loop order in QED comes out to be $B_b(0) = -226.27$ at $q_\perp^2 \to 0$. As expected, the helicity-flip fermion form factor $B_f(q^2)$ of the graviton coupling to the fermion constituent at one-loop order in QED comes out to be $B_f(0) = 226.27$ at $q_\perp^2 \to 0$ leading to the cancellation of the graviton couplings to the boson and fermion constituent. The Pauli form factor for $q_\perp^2 \to 0$ comes out to be $F_2(0) = 233.745$. In Fig. 1 we have presented the results for the gravitational form factors as a function of $q^2$ in the simulated QED. From plot it is clear that $B_f(q^2)$ and $B_b(q^2)$ exhibit opposite behaviour which leads to the vanishing result for gravitomagnetic moment.

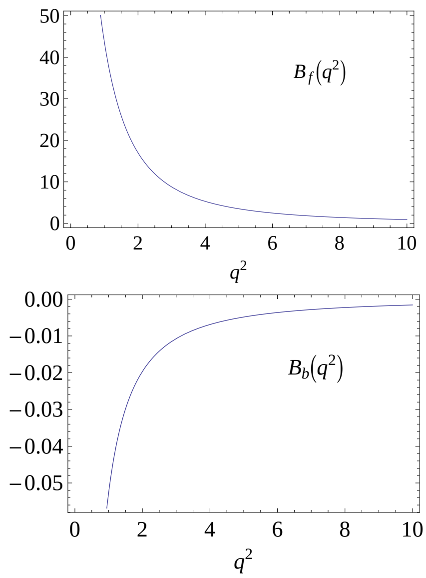


FIG. 1: Helicity-flip gravitational form factors for simulated model in QED.

## Conclusions

We have presented the results for the gravitational form factors as a function of $q^2$ in the simulated QED. We have also studied the Pauli form factor for both models which is obtained from the spin-flip matrix element of the $J^+$ current and when we compare the behaviour of $F_2(q^2)$ in both models, we find that due to the connection with the $x$ moments of chiral conserving and chiral flip form factors appearing in the deep virtual compton scattering, their behaviour is almost same. This fact can perhaps can be substantiated by future experiments in DVCS measurements of the spin flip form factors. New experiments aimed at measuring the anomalous gravitomagnetic moment would also provide us with valuable information on the hadron structure.

## References


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