Masses and electromagnetic decay constants of ground and radially excited states of $c\bar{c}$ and $b\bar{b}$ pseudoscalar mesons

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Introduction: Heavy flavor mesons are of the type $Q\bar{Q}$, or $Q\bar{q}$, in which one of the quarks belongs to a heavy flavour such as c or b. Investigation of properties of these mesons gives a lot of information about heavy quark dynamics. Further, these studies should provide an important tool for exploring the structures of these simplest bound states in QCD, and for studies on non-perturbative behavior of strong interactions. In this work we study the mass spectrum and leptonic decay constants of ground state (1S) and radial excitations (2S and 3S) of equal mass heavy pseudoscalar mesons such as η_c and η_b comprising of quark composition$c\bar{c}$ and $b\bar{b}$. Such studies have become a hot topic in recent years, due to observation of many new states at various high energy accelerators at BABAR, Belle, CLEO and BES-III collaborations [1]. This has opened up new challenges in theoretical understanding of these heavy hadrons.

However, any model should not only describe the mass spectrum of these states, but also other observables like the decay widths [2], transition amplitudes etc., Now, at very short distances, a perturbative Coulomb potential with a strength proportional to the strong coupling constant, α_s is derived from the one-gluon-exchange interaction in QCD. However, at long distances, non-perturbative effects such as confinement interaction has to be considered. In this work we consider a phenomenological confinement potential [2] that simulates an effect of an almost linear confinement (~r) for heavy quark (c,b) sector, while retaining harmonic form (~r^2) for light quark (u,d) sector as is believed to be true for QCD. In this work we use BSE (in a 4x4 representation) under Covariant Instantaneous Ansatz (CIA) [2] to calculate first the mass spectrum of heavy pseudoscalar mesons for both the ground states (1S), as well as the first and second radially excited states, (2S) and (3S) of η_c and η_b. We then apply this framework to study leptonic decays of all these states, which proceed through the coupling of neutral weak current to the two-quark loop.

Mass spectrum: The 4D BSE for a $q\bar{q}$ system, written in a 4X4 representation of the 4D BS wave function ψ(P,q) of a hadron with external momentum P and internal momentum q, and with interaction kernel K(q,q'), is:

$$-i(2\pi)^4 S_F^{-1}(p_1)\psi(P,q)S_F^{-1}(-p_2) = \int d^4q' K(q,q')\psi(P,q') \,. \quad (1)$$

We decompose the internal momentum of the hadron q_μ as the sum of two parts, the transverse component, $\hat{q}_\mu = q_\mu - \frac{q.P}{P^2}P_\mu$, which is orthogonal to total hadron momentum P_μ, and the longitudinal component, $\sigma P_\mu = \frac{q.P}{P^2}P_\mu$, which is parallel to P_μ. Following a sequence of steps, we obtain the mass spectral equation [3] as:

$$\left(\frac{M^2}{4} - m^2 + \frac{C_0}{\omega_0^2}\beta^4\right) = 2\beta^2\left(N + \frac{3}{2}\right); \quad (2)$$

N= 0,1,2,…, with $\beta^2 = (m\omega_0^2)^{\frac{1}{2}}$.

Decay constants: The decay constant f_P for P-meson is defined as [3-5],

$$if_P P_\mu =< 0|\bar{Q}i\gamma_\mu\gamma_5 Q|P>, \quad (3)$$

where f_P can be evaluated through the loop diagram which gives the coupling of two-quark loop to the axial vector current, and can be expressed as a loop integral,

$$f_P P_\mu = \sqrt{3}\int d^4 q Tr[\psi(P,q)i\gamma_\mu\gamma_5]. \quad (4)$$

Results and Discussions: The numerical calculation of mass spectrum and decay constants of was done using Mathematica. The input parameters of our model were fixed as: C_0=0.1, ω_0=0.01GeV., Λ=0.2GeV., and the input quark masses, m_c=1.53GeV., and m_b= 4.9GeV. The results of masses and the decay constants are given in Tables I and II respectively below.

Table I: Masses (in GeV.) of ground (1S) and radially excited (2S and 3S) states of η_c and η_b mesons along with data

	BSE-CIA	Expt.[1]
η_c(1S)	2.996	2.980∓.0012
η_c(2S)	3.027	3.637∓.004
η_c(3S)	3.0867	-
η_b(1S)	9.6113	9.3909∓.0028
η_b(2S)	9.6377	-
η_b(3S)	9.6641	-

The results obtained for masses of ground states are: M(η_c(1S))=2.996GeV.(Expt.=2.980∓.0012GeV.[1]) and M(η_b(1S))=9.611GeV. (Expt.=9.391∓.0028GeV.[1]) and thus are in good agreement with data. However the mass of η_c(2S) state is a bit lower at 3.027GeV.(Expt.=3.637GeV.[1]), than the corresponding experimental value for the same. Our predictions [3] of the masses of $\eta_c(3S)$ and radially excited states of η_b (shown in Table 1) are somewhat lower than the predictions of other models, though these states have not yet been observed in experiments so far.

Table II: Electromagnetic decay constants f_P (in GeV.) of ground (1S) and excited (2S, 3S) states of η_c and η_b mesons along with data.

	BSE-CIA	Expt.[1]
η_c(1S)	0.3472	0.335∓.075
η_c(2S)	0.2786	-
η_c(3S)	0.2487	-
η_b(1S)	0.8007	-
η_b(2S)	0.6421	-
η_b(3S)	0.5728	-

The leptonic decay constants f_P calculated for these all states are given in Table II. It is seen that our calculated f_P value for ground state,$f_{\eta_c(1S)}$=.3472GeV(Expt.=.335∓.075GeV[1]) and is within the error bars of the data. The f_P values of radially excited (2S and 3S) states of η_c and η_b are listed in Table II, though the experimental data for these states is not yet available. It is observed that the decay constants keep decreasing as one goes from (1S) to (3S) states for both η_c and η_b mesons, which is in conformity with other models.

References:

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