Phase Transition Study in Self-affine Scaling Scenario

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The study of quark-hadron phase transition has been a hot point in both particle physics and nuclear physics for more than a decade. Among the various methods the Levy stable law [1] is used to detect the existence of possible phase transition in hadronization process. This law is characterized by the Levy stability index μ which takes value in the range [0, 2]. A thermal phase transition occurs when $\mu \langle 1$ and when $\mu \rangle 1$, there is a non-thermal phase transition during the cascading process. According to Ginzburg-Landau (GL) theory for second order phase transitions in hadronization process [2], the anomalous fractal dimension (d_q) follows the relation $\frac{d_q}{d_2} = (q-1)^{\nu-1}$, where scaling exponent ν = 1.304, a universal quantity. If the measured value of ν is close to this critical value, then a second order quark-hadron phase transition is expected. To get more information about the inner dynamics of the particle production in high-energy interactions, the phase transition and its dependence on target excitation has been studied thoroughly using the Levy stability analysis of the produced pions for $\pi^- - AgBr$ interactions at 350 GeV in two dimensional $(\eta - \phi)$ phase space under self-affine scenario. Finally GL theory is used to search for the second order quark-hadron phase transition.

Factorial moment of order q is defined as [3] $F_q(\delta x_1, \delta x_2)$

$$= \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m (n_m - 1)........(n_m - q + 1)\rangle}{\langle n_m \rangle^q},$$

n_m is the multiplicity in the m^{th} cell. M is the number of two-dimensional cells into which the considered phase space has been divided. Here M_1 is the number of bins along x_1 direction and M_2 is the number of bins along x_2 direction. The shrinking ratios along x_1 and x_2 directions are characterised by a parameter $H = \ln M_1 / \ln M_2$ where $0 < H \le 1$ is called the Hurst exponent. H <1 signifies that the phase spaces are divided anisotropically, consequently the fluctuations are self affine in nature. From the power law dependence of factorial moment on the cell size as cell size approaches to zero, the index α_q is obtained from a linear fit of the form $\ln\langle F_q \rangle = \alpha_q \ln M + a$, where a is a constant. Now a quantity β_q is defined as $\beta_q = \frac{d_q}{d_2}(q-1)$ and β_q is related to Levy index (μ) by the equation $\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{q^\mu - q}{2^\mu - 2}$. According to GL theory $\beta_q = (q-1)^\nu$ with ν = 1.304 as the critical exponent.

In order to reduce the effect of non-flat average distribution, the cumulative variables X_η and X_ϕ are used instead of η and ϕ. The new "cumulative" variable X_z is related to the original single- particle density distribution $\rho(z)$ as, $X_z = \int_{z_{\min}}^{z} \rho(z')\partial z' / \int_{z_{\min}}^{z_{\max}} \rho(z')\partial z'$, where $z_{\min}$ and $z_{\max}$ are the two extreme points of the distribution. In the X_η - X_ϕ space we divided the region [0, 1] into M_η & M_ϕ bins respectively. The partitioning was taken as $M_\eta = M_\phi{}^H$. We choose the partition number along ϕ direction as M_ϕ = 2, 3, ……, 20. The (X_η - X_ϕ) space is divided into M = $M_\eta \times M_\phi$ cells and calculation is done in each bin independently. In the (X_η - X_ϕ) phase space the anisotropic behavior of pions is best revealed at H = 0.3. For this H value the plot of $\ln\langle F_q \rangle$ as a function of $\ln M$ gives the value of α_q. Using the values of α_q , values of β_q are calculated. It is observed that the parameter β_q increases with increasing order of moments. This indicates the fact that charged particle density distribution has multifractal structure. Therefore, we can say that hadrons in the final state are produced as a result of a self-similar cascade mechanism. From values of β_q the Levy index obtained for the $\eta - \phi$ space is μ = 0.468 ± 0.005 which is within the permissible limit 0≤ μ ≤2. Here μ 〈 1 would have indicated a thermal phase transition of second order.

For studying target excitation dependence we have divided the data set for pions into three sets , 0 ≤ n_g ≤2, 3≤ n_g ≤5, 6≤ n_g ≤13, depending on the number of grey tracks (n_g).The sets correspond to different degrees of target excitation. In the self-affine space the Levy index analysis is repeated for the three target excitation data sets. It is observed that the parameter β_q increases with increasing order of moments revealing multifractal pattern of produced pions in different n_g intervals. The values of μ are calculated and listed in Table 1.We get μ 〈 1 for three target excitation data sets indicating a second order thermal phase transition (interspersed in the cascading process) with a large latent heat, and thus may serve as a possible indication of QGP being formed. Moreover, the values of Levy indices (μ) vary consistently with degrees of target excitation.

Again according to the GL theory the values of ν for full data set and for three n_g intervals are calculated and are listed in Table 1. From the table it is observed that the values of ν are significantly different from the critical value of ν making the GL description inappropriate and second order phase transition can most likely be ruled out.

Table 1: Values of different parameters (β_q, μ and ν)

n_g	H	μ	ν
All n_g	0.3	0.468 ± 0.005	1.110 ± 0.002
0 ≤ n_g ≤2	0.3	0.542 ± 0.002	1.131 ± 0.004
3 ≤ n_g ≤ 5	0.7	0.478 ± 0.012	1.112 ± 0.007
6 ≤ n_g ≤ 13	0.3	0.425 ± 0.012	1.098 ± 0.007

References

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