Net-baryon number fluctuations using Tsallis Statistics

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New results of the STAR collaboration show the energy dependence of the moments (mean M, variance σ, skewness S and kurtosis κ) and their products for net-proton multiplicity distribution at RHIC energies [1]. The product of the moments $S\sigma$ and $\kappa\sigma^2$ are related to the ratios of susceptibilities (χ) associated with the baryon number conservation. The recent STAR measurements of $S\sigma$ and $\kappa\sigma^2$ show significant deviations from the predictions of the Skellam distribution (where $\kappa\sigma^2$ should be unity) at all energies indicating presence of large non-statistical fluctuations. The particle production in heavy ion collisions at relativistic energies is well described in terms of the hadron resonance gas (HRG) model. The success of HRG model would mean that thermal system which might have gone through a possible phase transition has (nearly) equilibrated. It is believed that if the thermal system has retained some memory of the phase transition with finite correlation length at freeze out, it must be reflected in the higher moments of the conserved quantities. Therefore, the study of fluctuations in various conserved quantities like: net-baryon number through the higher moments using HRG model is expected to provide a baseline to observe the deviation in experimental observables, which may indicate the presence of non statistical fluctuations, if any. It has been realized that particle production in heavy ion and proton-proton collisions at RHIC and LHC energies can be described successfully using a power law type distribution rather than the exponential one. Therefore, the Tsallis distribution function is being used for particle production with nonextensive parameter q such that it approaches Boltzmann distribution in the limit $q \to 1$ [2].

In the present work, we show that the HRG-Tsallis model with a temperature dependent nonextensive q parameter reproduces the energy dependence of $S\sigma$ and $\kappa\sigma^2$ for most peripheral collisions as well as $S\sigma$ for central collisions. However, the energy dependence of $\kappa\sigma^2$ of central collision deviate significantly from the HRG-Tsallis model predictions particularly at energies 19.6 GeV and 27 GeV. We argue here that the predictions of HRG-Tsallis characterized by a temperature dependent q parameter should be taken as the baseline to study (experimentally) fluctuations of dynamical origin if any, which is still not contained in the Tsallis non-extensive thermodynamics.

The Tsalli's form of Fermi-Dirac (FD) and Bose-Einstein (BE) distributions can be written as,

$$f = \frac{1}{\exp_q \frac{(E-\mu)}{T} \pm 1} \qquad (1)$$

where "+" and "-" signs are used for fermions and bosons respectively and $\exp_q(x)$ is given by,

$$\exp_q(x) = \begin{cases} [1+(q-1)x]^{1/(q-1)} & \text{if} \quad x > 0 \\ [1+(1-q)x]^{1/(1-q)} & \text{if} \quad x \leq 0 \end{cases}$$

where $x = (E-\mu)/T$. The above distribution approaches standard FD and BE distributions in the limit $q \to 1$. Using above nonextensive distribution function, the average number density (n_q) can be calculated. Note that the exponent q in f_i has been introduced as a constraint for thermodynamical consistency.

Since the average density and the pressure P are shown to be thermodynamically consistent i.e. $n_q = \frac{\partial P}{\partial \mu}$, we can define a generalized

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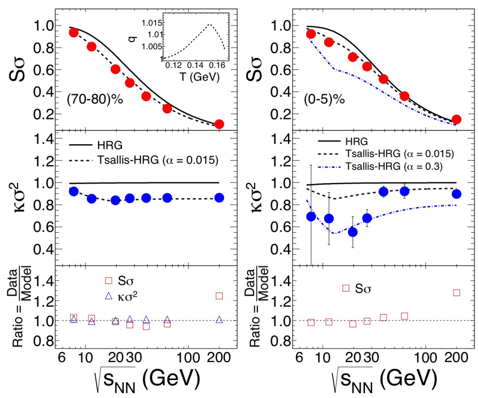

FIG. 1: (Color online) The energy dependence of product of moments, $S\sigma$ and $\kappa\sigma^2$ of net-baryon distribution calculated using standard HRG (solid curve) and Tsallis-HRG with different α (dotted curve) for peripheral and central collisions are shown. The filled circles are STAR data points. The bottom panel shows the ratio of data and our model calculation. The figure inset at the top of the left panel shows the temperature dependence of the q parameter.

susceptibility as,

$$\chi_q^n = \frac{\partial^n[P(T_f,\mu)]}{\partial\mu^n}|_T = \frac{\partial^{n-1}[n_q(T_f,\mu)]}{\partial\mu^{n-1}} \quad (2)$$

Using Eq.2, we have estimated $S\sigma$ (χ^3/χ^2) and $\kappa\sigma^2$ (χ^4/χ^2) for net-baryon density within STAR detector acceptance.

We parametrized the freeze-out temperatures and chemical potentials in the same way, as they are used in Ref. [3] for most central collision. For peripheral collision, we extract these parameters from STAR preliminary data for most peripheral (70-80)% centrality collisions. Note that we use the same parametrization irrespective of whether the collision is central or peripheral except different centrality will have different chemical freeze-out parameters.

Left panel of Figure 1 , shows the energy dependence of $S\sigma$ (top panel) and $\kappa\sigma^2$ (middle panel) estimated using parameters for most peripheral (70-80)% centrality collisions. As can be seen, the HRG results ($\alpha = 0$) do not explain the experimental values [1], and $\kappa\sigma^2$ in HRG model is always close to unity where as data points are about 20% below the HRG values. On the other hand, HRG-Tsallis with a nominal $\alpha = 0.015$ (corresponds to a maximum q value of 1.02) can explain both $S\sigma$ and $\kappa\sigma^2$ extremely well. The corresponding results for most central (0-5)% centrality collisions are shown in the right panel of Fig.1. Also for the central collisions, the HRG predictions do not explain the experimental data.The excellent agreement between data and model predictions can be seen from the ratio plot in the bottom panel of Fig.1. Details of parametrization can be found in Ref. [4].

In conclusion, we have studied the energy dependence of the fluctuations of net-baryon productions through higher moments namely $S\sigma$ and $\kappa\sigma^2$ using HRG with Tsallis non-extensive distribution function. When the non-extensive parameter q is close to unity, the moments obtained using HRG-Tsallis model can be interpreted as the weighted average of the moments estimated using many Boltzmann distribution corresponding to the distribution of temperatures over the whole phase space. It is shown that this HRG-Tsallis model can explain the energy dependence of $S\sigma$ and $\kappa\sigma^2$ measurements of recently published STAR data corresponding to the most peripheral collisions which is otherwise impossible to explain using normal HRG model which predicts $\kappa\sigma^2$ close to unity for net-baryon productions.

Acknowledgments

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