Multifractal detrended fluctuation analysis of ^{16}O-Ag/Br interaction at 60A GeV

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Introduction

In recent years the detrended fluctuation analysis (DFA) method **[1]** has become a widely used technique for the determination of (mono-) fractal scaling properties and the detection of long-range correlations in noisy, nonstationary time series **[2]**. One reason to employ the DFA method is to avoid spurious detection of correlations that are artifacts of nonstationarities in the time series. It has successfully been applied to diverse fields such as DNA sequences, heart rate dynamics, neuron spiking, human gait, long-time weather records, cloud structure, geology, ethnology, economics time series, and solid state physics. For many series, the scaling behavior is more complicated, and different scaling exponents are required for different parts of the series. This occurs, e.g., when the scaling behavior in the first half of the series differs from the scaling behavior in the second half. In even more complicated cases, such different scaling behavior can be observed for many interwoven fractal subsets of the time series. In this case a multitude of scaling exponents is required for a full description of the scaling behavior, and a multifractal analysis must be applied.

In general, two different types of multifractality in time series can be distinguished: (i) Multifractality due to a broad probability density function for the values of the time series. In this case the multifractality cannot be removed by shuffling the series. (ii) Multifractality due to different long-range (time-) correlations of the small and large fluctuations. In this case the probability density function of the values can be a regular distribution with finite moments, e.g. a Gaussian distribution. The corresponding shuffled series will exhibit nonmultifractal scaling, since all long-range correlations are destroyed by the shuffling procedure. If both kinds of multifractality are present, the shuffled series will show weaker multifractality than the original series. The simplest type of multifractal analysis is based upon the standard partition function multifractal formalism, which has been developed for the multifractal characterization of normalized, stationary measures **[3]**. Unfortunately, this standard formalism does not give correct results for nonstationary time series that are affected by trends or that cannot be normalized. Thus, in the early 1990s an improved multifractal formalism has been developed, the wavelet transform modulus maxima (WTMM) method **[4]**, which is based on wavelet analysis and involves tracing the maxima lines in the continuous wavelet transform over all scales. Here, we propose an alternative approach based on a generalization of the DFA method. This multifractal DFA (MF-DFA) does not require the modulus maxima procedure, and hence does not involve more effort in programming than the conventional DFA.

Here we analyzed MF-DFA for ^{16}O-Ag/Br interaction at 60 A GeV **[5]** in pseudorapidity phase space to get an idea of the underlying phase space structure of particle production in that interactions.

Methodology

The multifractal generalization of this procedure MFDFA can be briefly sketched as follows. First, for a given time series {x(i); i=1……N} on a compact support, one calculates the integrated signal profile $Y(j) = \sum_{i=1}^{j}(x(i) - \langle x \rangle), j = 1 \dots N$. Where, <…> denotes averaging over the time series, and then one divides it into M_n segments of length n (n<N) starting from both the beginning and the end of the time series (i.e., $2M_n$ such segments total). Each segment ν has its own local trend that can be approximated by fitting an l^{th} order polynomial $P_\nu^{(l)}$ and subtracted from the data; next, the variances for all the segments ν and all segment lengths n must be evaluated $\boldsymbol{F^2(\nu, n) = \frac{1}{n}\sum_{j=1}^{n}\{Y[\nu - 1)n + j] - P_\nu^{(l)}(j)\}^2}$.

Finally, $\boldsymbol{F^2(\nu,n)}$ is averaged over *v's* and the q^{th}-order fluctuation function is calculated for all possible segment lengths *n*:

$$\boldsymbol{F_q(n) = \left(\frac{1}{2M_n}\sum_{\nu=1}^{2M_n}[F^2(\nu,n)]^{q/2}\right)^{1/q},\ q\epsilon R} \text{--}\mathbf{(1)}$$

The key property of $F_q(n)$ is that for a signal with fractal properties, it reveals power-law scaling within a significant range of *n*,

$$\boldsymbol{F_q(n) \sim n^{h(q)}} \text{------------------------------------}\mathbf{(2)}$$

The result of the MFDFA procedure is the family of exponents *h(q)* (called the generalized Hurst exponents). For very large scales, *n>N/4*; $F_q(n)$ becomes statistically unreliable because the number of segments M_n for the averaging procedure becomes very small. Thus, we usually exclude scales *n>N/4* from the fitting procedure to determine *h(q)*. The value of *h(0)*, which corresponds to the limit *h(q)* for q → 0, cannot be determined directly using the averaging procedure in Eq. (1) because of the diverging exponent. Instead, a logarithmic averaging procedure has to be employed,

$$\boldsymbol{F_0(n) \equiv \exp\left\{\frac{1}{4M_n}\sum_{\nu=1}^{2M_n} ln\,[F^2(\nu,n)]\right\} \sim n^{h(0)}}$$

For an actual multifractal signal, form a decreasing function of *q*, while for a monofractal *h(q)*=const **[1]**.

Results & Discussion

We have shown the variation of Logarithmic variation of generalized fluctuation functions $\boldsymbol{F_q(n)}$ with log of scale $\boldsymbol{n}$ for q=±4, ±3, ±2,±1, ±0.9,±0.8,±0.7,±0.6,±0.5,±0.4,±0.3,±0.2,-1.53, -3.4,+0.024,+0.096,+0.167,+0.265,+1.33,+1.407, in **Fig.1**. On large scales *n*, we observe the expected power-law scaling behavior according to Eq. (2), which corresponds to straight lines in the log–log plot.

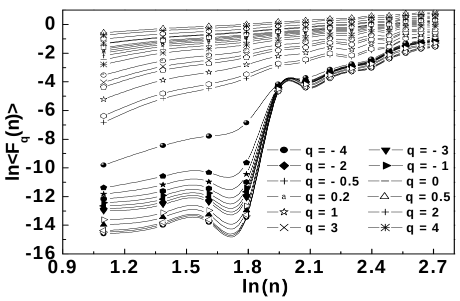

Fig.1

In **Fig.2**, the scaling exponents *h(q)* determined from the slopes of these straight lines are shown versus *q*. Clearly, q dependency is observed for *h(q)*.

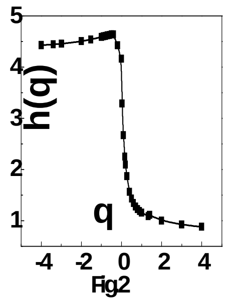

Fig2

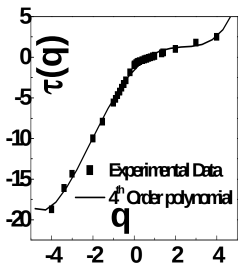

Fig.3

Scaling exponent, **[1]** *τ(q) =qh(q)-1* has also been plotted in **Fig.3**. We fitted the experimental points with 4^{th} order polynomial and it is clear that *τ(q)* is showing a strong nonlinear dependency with *q*.

From the above analyses it has been found that *h(q)* is showing a q dependency which may be a signal of multifractality of the phase space structure of particle production. At the same time we also observed a nonlinear q dependency of τ(q) indicating multifractality of phase space structure. However, a definite conclusion cannot be drawn from the above analyses about the phase space structure, this topic needs serious investigation.

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