Superfluid neutron stars with isospin dependent entrainment effect

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Rapid cooling of Cassiopeia A[1, 2] as well as pulsar glitches [3, 4] might be explained by considering superfluid neutrons in a neutron star core. The neutron stars are described by the two fluid model where one fluid is superfluid neutrons and the other fluid called the proton fluid represents the charge neutral component made of protons and electrons. When one fluid interpenetrates through the other,the momentum of one fluid is proportional to the linear combination of velocities of both fluids. This effect is known as entrainment. We study the entrainment effect in a neutron star interior within the two-fluid formalism using a relativistic model where baryon-baryon interaction is mediated by exchange of σ,ω and ρ mesons. This model of strong interaction also includes scalar self interaction. The starting point of the superfluid formalism is the master function $-\Lambda(n^2, p^2, x^2)$ that is a function of scalars which are obtained neutron and proton number density currents. The stress-energy tensor in terms of the master function is written as [5, 6] $T^{\mu}_{\nu} = \Psi\delta^{\mu}_{\nu} + n^{\mu}\mu_{\nu} + p^{\mu}\chi_{\nu}$ where $\Psi = \Lambda - n^{\rho}\mu_{\rho} - p^{\rho}\chi_{\rho}$ is the generalized pressure and

$$\begin{aligned} \mu_{\nu} &= \mathcal{B}n_{\nu} + \mathcal{A}p_{\nu} \ , & (1) \\ \chi_{\nu} &= \mathcal{A}n_{\nu} + \mathcal{C}p_{\nu} \ , & (2) \end{aligned}$$

μ_{ν} and χ_{ν} are neutron and proton momentum covectors. The entrainment effect is present due to the term $\mathcal{A}$ in above equations. Now we relate the neutron and proton number density currents to the mean particle fluxes of neutrons and protons along the z direction in relativistic mean field model[7]. Comer and Joynt [5] calculated the entrainment effect using the σ-ω model. So the symmetry energy effect is absent from the system. We include ρ mesons and scalar self coupling terms in the Lagarangian [7]. The Lagrangian density for nucleon-nucleon interaction is given by[8]

$$\begin{aligned} \mathcal{L}_B &= \sum_{B=n,p} \bar{\Psi}_B(i\gamma_{\mu}\partial^{\mu} - m_B + g_{\sigma B}\sigma \\ &\quad - g_{\omega B}\gamma_{\mu}\omega^{\mu} - g_{\rho B}\gamma_{\mu}\mathrm{t}_B \cdot \boldsymbol{\rho}^{\mu})\Psi_B \\ &\quad + \frac{1}{2}\left(\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^2\sigma^2\right) - U(\sigma) \\ &\quad - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} \\ &\quad + \frac{1}{2}m_{\rho}^2\boldsymbol{\rho}_{\mu} \cdot \boldsymbol{\rho}^{\mu} \ . \qquad (3) \end{aligned}$$

We calculate the equations of motion for mesons field in the mean field approximation and averaged stress energy tensor in a zero momentum frame of neutrons. In the slow rotation approximation, we get the values of $\Psi|_0, \Lambda|_0, \mu|_0, \chi|_0$, $\mathcal{A}|_0, \mathcal{B}|_0$ and $\mathcal{C}|_0$ from Ref.[7] where subscript "0" stands for quantities calculated in the static limit and $c_{\sigma}^2 = (g_{\sigma}/m_{\sigma})^2$, $c_{\omega}^2 = (g_{\omega}/m_{\omega})^2$ and $c_{\rho}^2 = (g_{\rho}/m_{\rho})^2$. Now the entrainment parameter (ϵ_{mom}) in the zero momentum frame of neutrons can be calculated in terms of coefficients in momentum covectors in the two fluid formalism from the following relation [5, 7],

$$\epsilon_{mom} = \frac{m}{n}\frac{\mathcal{A}|_0}{(\mathcal{B}|_0\mathcal{C}|_0 - \mathcal{A}|_0^2)} \ . \qquad (4)$$

Here we consider a beta-equilibrated neutron star matter made of neutrons, protons and electrons. We perform this calculation for the NL3 and GL parameter sets as shown in Table 1. For the calculation of entrainment we choose neutron star configurations which are just below the maximum masses in both case. In case of the GL set, we consider a neutron star mass of 2.37 $M_{\odot}$ corresponding to the central value of neutron wave number

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TABLE I: Nucleon-meson coupling constants in the GL and NL3 sets are taken from Ref.[7–9]. All the parameters are in fm^2, except b and c which are dimensionless.

	c_σ^2	c_ω^2	c_ρ^2	b	c
GL	12.684	7.148	4.410	0.005610	-0.006986
NL3	15.739	10.530	5.324	0.002055	-0.002650

$k_n(0) = 2.71 fm^{-1}$. The radius of the neutron star is 11.09 km. Similarly, in the other case with NL3 set, we find a neutron star having maximum mass 2.82 $M_\odot$ and radius 13.17 km. The radial profile of the entrainment parameter in the zero momentum frame for the GL and NL3 sets are shown in Figure 1. The entrainment parameter in both cases remains constant in the core and drops rapidly at the surface. Moreover, in both cases, the entrainment effect is strong at higher baryon densities in the core whereas this effect diminishes sharply at lower densities towards the surface. For the calculation of the entrainment parameter without ρ meson, we obtain the radial profile of the entrainment parameter in a neutron star of 2.33 $M_\odot$ and radius 10.96 km. It is evident from Fig. 1 that the inclusion of ρ in the calculation strongly enhances the entrainment parameter.

Furthermore, We investigate the role of the isospin dependent entrainment on the mass, shape and kepler limit of of slowly rotating neutron stars. It is found from Fig. 2 that the Kepler limit is modified due to the isospin dependent entrainment effect.

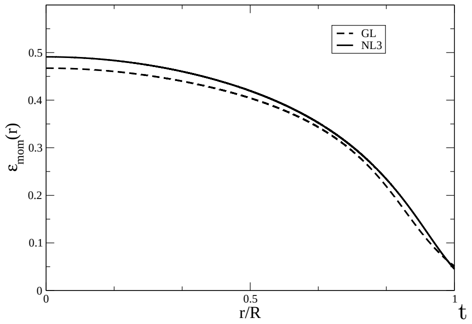

FIG. 1: Entrainment parameter in the RMF model with ρ meson in the zero momentum frame of neutrons is plotted as a function of radial distance in a neutron star .

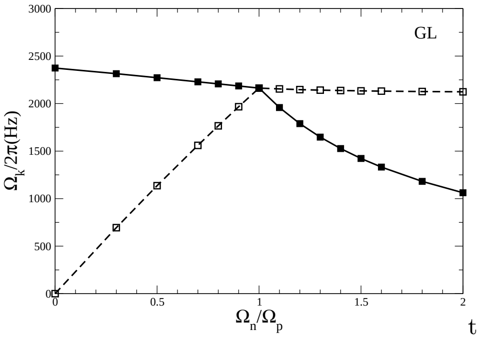

FIG. 2: The mass-shedding (Kepler) limit is shown as a function of relative rotation rate Ω_n/Ω_p for the GL set . The solid squares show the allowed rotation rate of proton(Ω_p) and the open squares show the allowed rotation rate of neutron(Ω_n). The Kepler frequency is the largest of the two.

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