# Information content of nuclear masses: A covariance analysis



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Constraining quantitatively the symmetry energy $C_v^0$ of nuclear matter at saturation density $\rho_0$ and its density slope $L_0(= 3\rho_0 \frac{\partial C_v(\rho)}{\partial \rho}|_{\rho_0})$ has been a major focus of attention in present-day nuclear physics. They are fundamental parameters in influencing the binding energies and stability of atomic nuclei. They are also of seminal importance in astronomical context. In a controlled finite-range droplet model (FRDM) from a fit of the observed nuclear masses, the uncertainty in the value of $C_v^0$ was found to be 32.5±0.5 MeV [1]. Studying meticulously the double differences of symmetry energies estimated from experimental nuclear masses its value was evaluated to be $32.10 \pm 0.31$ MeV [2]. Explorations on the symmetry slope $L_0$, however, show wide variations (20 - 120 MeV) [3].

Analysis of data on the nuclear masses in macroscopic models seems to contain the fluctuations in the value of $L_0$ somewhat better. Within the FRDM [1] , the value of $L_0$ was found to be 70±15 MeV. Consistent with the values of $C_v^0$ and surface symmetry energy coefficient $C_s$ (as determined from double differences of symmetry energies estimated from experimental nuclear masses [2] to be 58.91±1.08 MeV), a recent effort aided by microscopic calculations on the neutron-skin thickness $\Delta r_{np}$ of heavy nuclei [4] gives $L_0 = 59 \pm 13$ MeV, where, $\Delta r_{np}$ is the difference between the rms radii for density distributions of the neutrons and protons in a nucleus. Energy density functionals (EDF) in microscopic mean field models, parametrized to reproduce the binding energies of nuclei along with some other specific nuclear observables such as charge radii do not, however, display such constraints on the symmetry elements of nuclear matter.

In this contribution we show that the apparent irreconciliation can be eased if some highly asymmetric even-even spherical nuclei are additionally included in the fitting protocol of the optimization of the EDFs in microscopic mean field models. The relativistic mean-field model (RMF) is chosen as the vehicle for the realization of our goal. For this purpose, two models (model-I and model-II) corresponding to different sets of fit-data are constructed. In model-I the binding energies and charge radii of some standard set of nuclei ($^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{68}$Ni, $^{90}$Zr, $^{100}$Sn, $^{116}$Sn, $^{132}$Sn, $^{144}$Sm and $^{208}$Pb) are taken as fit-data. In model-II, we have the same set of experimental observables, but with the addition of the binding energy difference $\Delta B$ of ($^{24}$O,$^{16}$O) and of ($^{30}$Ne,$^{18}$Ne) [5], where, $\Delta B(X,Y) = B(X) - B(Y)$.

The parameters of model-I and model-II are obtained by optimizing the objective function $\chi^2(\mathbf{p})$ as given by,

$$\chi^2(\mathbf{p}) = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left( \frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}(\mathbf{p})}{\Delta \mathcal{O}_i} \right)^2, \quad (1)$$

where, $N_d$ and $N_p$ are the number of experimental data points and the number of fitted parameters, respectively. $\Delta\mathcal{O}_i$ is the adopted error and $\mathcal{O}_i^{exp}$ and $\mathcal{O}_i^{th}(\mathbf{p})$ are the experimental and the corresponding theoretical values for a given observable. Once the optimized parameter set is obtained the correlation coefficient between two quantities $\mathcal{A}$ and $\mathcal{B}$, which may be a parameter as well as an observable, can be evaluated within the covariance analysis as,

$$c_{\mathcal{AB}} = \frac{\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}}}{\sqrt{\overline{\Delta\mathcal{A}^2}\ \overline{\Delta\mathcal{B}^2}}}, \quad (2)$$

where, covariance between $\mathcal{A}$ and $\mathcal{B}$ is expressed as,

$$\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}} = \sum_{\alpha\beta} \left( \frac{\partial \mathcal{A}}{\partial \mathrm{p}_\alpha} \right)_{\mathbf{p_0}} \mathcal{C}_{\alpha\beta}^{-1} \left( \frac{\partial \mathcal{B}}{\partial \mathrm{p}_\beta} \right)_{\mathbf{p_0}}. \quad (3)$$

Here, $\mathcal{C}_{\alpha\beta}^{-1}$ is an element of inverted curvature matrix given by,

$$\mathcal{C}_{\alpha\beta} = \frac{1}{2} \left( \frac{\partial^2 \chi^2(\mathbf{p})}{\partial \mathrm{p}_\alpha \partial \mathrm{p}_\beta} \right)_{\mathbf{p}_0}, \quad (4)$$

where, $\mathbf{p}_0$ represents the optimized set of parameters. The square of the error, $\overline{\Delta\mathcal{A}^2}$ in $\mathcal{A}$ can be computed using Eq. (3) by substituting $\mathcal{B} = \mathcal{A}$.

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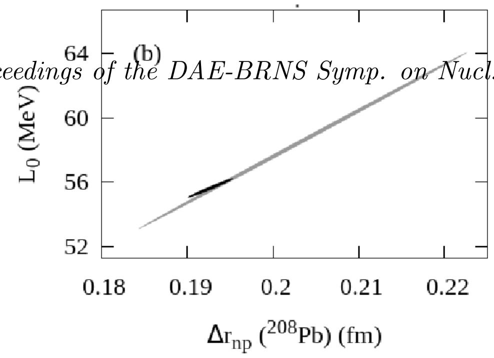


FIG. 1: Confidence ellipsoids between $L_0$ and the $\Delta r_{np}$ in the $^{208}$Pb nucleus for Model-I (elongated) and Model-II (contracted).

In Fig. 1 confidence ellipsoids between $L_0$ and the $\Delta r_{np}$ in the $^{208}$Pb nucleus are displayed for reasonable domain of parameters [5]. It clearly shows a strong correlation between them as suggested by Droplet Model [3]. Contracted shape of the ellipsoid for Model-II depicts that $L_0$ and the $\Delta r_{np}$ are better constrained in Model-II in comparison to Model-I. In Fig. 2 we have looked into

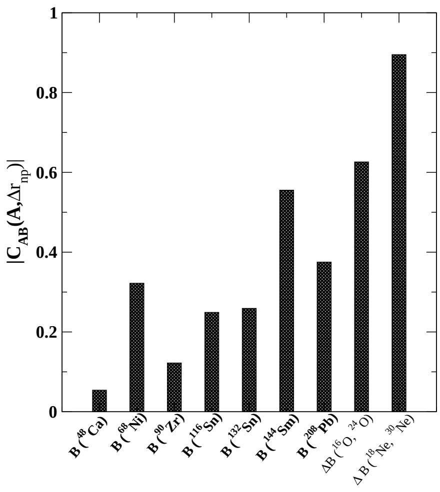


FIG. 2: Correlation coefficients $c_{AB}$ between different asymmetric nuclei and $\Delta r_{np}$ in the $^{208}$Pb nucleus for Model-II.

the correlation coefficients $c_{\rm AB}$, $A$ being the binding energies of asymmetric nuclei and $B$ the $\Delta r_{np}$ in the $^{208}$Pb nucleus. The correlation coefficient $c_{\rm AB}$ was found to be 0.626 and 0.895 when $A$ is taken as the binding energy difference $\Delta B(^{24}\mathrm{O},^{16}\mathrm{O})$ and $\Delta B(^{30}\mathrm{Ne},^{18}\mathrm{Ne})$ respectively. It appears that the correlation coefficient $c_{\rm AB}$ between the binding energy of a nucleus and the neutron-skin thickness in $^{208}$Pb is large for $\Delta B(^{24}\mathrm{O},^{16}\mathrm{O})$ and $\Delta B(^{30}\mathrm{Ne},^{18}\mathrm{Ne})$ in comparison to other nuclei considered.

Nuclear matter properties for model-I and model-II are compared in Table I. Errors on the entities describing the isoscalar behavior of nuclear matter (binding energy per nucleon $E/A$, incompressibility $K$, saturation density $\rho_0$ and effective mass $m^*/m$) are pretty much the same for both the models concerned. For model-II, however, a significant improvement ($\sim 50\%$) on the spread of parameters like $C_v^0$ and $L_0$, which describe the symmetry behavior of nuclear matter, is achieved over model-I.

TABLE I: The values of nuclear matter properties $E/A$, $K$, $C_v^0$ and $L_0$ (MeV); $\rho_0$ (fm$^{-3}$) and $^{208}$Pb (fm) along with statistical errors on them for the models I and II.

| Observable | model-I | model-II |
|---|---|---|
| $E/A$ | $-16.036 \pm 0.070$ | $-16.036 \pm 0.051$ |
| $K$ | $210.12 \pm 27.87$ | $209.64 \pm 28.52$ |
| $\rho_0$ | $0.150 \pm 0.003$ | $0.150 \pm 0.003$ |
| $m^*/m$ | $0.585 \pm 0.012$ | $0.585 \pm 0.010$ |
| $C_v^0$ | $32.03 \pm 3.08$ | $31.69 \pm 1.51$ |
| $L_0$ | $57.62 \pm 17.08$ | $55.63 \pm 7.00$ |
| $\Delta r_{np}$ ($^{208}$Pb) | $0.201 \pm 0.065$ | $0.193 \pm 0.030$ |

To summarize, we have tried to give a critical look on the question why the information content on the elements of symmetry energy do not get transported properly in the EDFs when they are parametrized to reproduce the binding energies of representative nuclei in a mean-field model. The asymmetry parameters are then less constrained. Working in the confines of RMF model, we find that better constraint can be achieved in the said symmetry elements if one includes, in the fitting protocol of the microscopic model binding energies of some highly asymmetric nuclei. The reduction in the uncertainties on the values of symmetry energy and its slope parameter by $\sim 50\%$ in comparison to those obtained without the inclusion of the additional nuclei in the fit data, points in this direction.

## References

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