Energy levels observed as $g, \beta_1, \beta_2, \beta_3, \gamma$ and $\gamma\gamma$ bands in some even nuclei support the rotational structure

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According to Bohr – Mottelson unified collective model [1] the low – lying levels of even Z even N nuclei in mass region A ~ 220 – 250 away from closed shells are expected to develop collective characteristics. The lowest levels are grouped into three K – bands. The lowest one based on the ground state forms the ground state rotational band (K = 0_1). Another band based on the axially symmetric vibrations of the nuclear core with K = 0_2 is called the β – vibrational band. The third one based on axially asymmetric vibrations of the nuclear core with K = 2 projection on the symmetry axis is called γ – vibrational band. In a rigid rotor model the rotation of the rigid asymmetric nucleus not only yields yrast (ground state rotational) band but also 2+, 3+, 4+, 5+,……….levels (anomalous rotational band or γ – band) as the value of the asymmetry parameter γ increases. Toki and Faessler have commented that the γ – band is one rotational band when the asymmetry parameter is small ($0^0 < \gamma < 15^0$), at larger γ – deformation one should take $R = 2_2^+, 4_2^+, 6_2^+ \ldots$ as one band and $R = 3_1^+, 5_1^+, 7_1^+ \ldots\ldots$as another band. This observation is based on the study of the quadrupole moments and the B (E2) values as a function of the γ – deformations [2]. The physical reason is attributed to a repulsive force between even spin members of ground band and even spin members of γ – band that gives up turn to $E4_2^+, E6_2^+$, $E8_2^+$ at $\gamma > 15^0$. The examples of collective excitations resulting in both oscillations and rotations are well established in the form of γ – soft model by Wilets – Jean [3] and γ – rigid model by Davydov – Filippov [4]. The energy gaps $\Delta E_1 = E3_1^+ - (E2_1^+ + E2_2^+)$ and $\Delta E_2 = E3_1^+ - (2E2_1^+ + E4_1^+)$ plays an important role in distinguishing the rotational and oscillatory structure of nucleus Viz. $\Delta E_2 >> \Delta E_1 \approx 0$ (In rotational nuclei) and $\Delta E_1 >> \Delta E_2 \approx 0$ (in Oscillatory nuclei). The energy ratio $R\left(\frac{4}{2}\right) = \frac{E4_1^+}{E2_1^+}$ is a good measure of the deformation of a nucleus with the variation in N and Z. it can assume value of 2.0 for the spherical vibrator to 3.33 for the deformed rotor in the ground state band. For a collective rotations based on the intrinsic axial vibration the value of R (4/2) is expected to be the same for β – band (K = 0_2) and ground state band (K = 0_1).

Thus in broader perspective, the present viewpoint emphasizes that whether the low – lying energy levels obey I (I+1) rule which is meant for rotational structure. Earlier efforts have been made to test the γ – band obeys this rule by employing the soft rotor formula [5]. Recently, the soft rotor formula was modified by Gupta et al; with the plea that the large value of softness parameter (α) and negative values of moment of inertia (θ) are unjustified [6]. The modified soft Rotor formula is given as-

$$E(I) = E_K + \frac{\hbar^2 I(I+1)}{2\theta_0(1+\alpha I)}$$

In the present work the modified soft rotor formula is employed to all observed low –lying energy levels denoted as g, β_1, β_2, γ_1 and γ_2 for ^{168}Er, ^{160}Yb and ^{234}U. The list of parameters used in the present work are tabulated in table – I . The calculated energies are presented in table – II along with the experimental ones.

The close values of energy levels in our calculations with experiment are sufficient proof of rotational structure of observed $g, \beta_1, \beta_2, \beta_3, \gamma_1$ and γ_2 levels. The different moment of inertia of nuclei (Table – I) in producing these rotational bands indicates that possibility of many axes rotations of the nucleus is not a dream.

Table – I

List of parameters used in the present work.

Bands	θ	α	E_K(MeV)
^{168}Er			
g	37.3	0.0046	0
β_1	50.2	0.0070	1.217
β_2	41.8	0.0055	0
β_3	50.8	0.0050	0
γ_1	39.4	-	0.0746
^{160}Yb			
g_1	9.0	0.185	0
g_2	1.62	0.756	5.093
^{234}U			
γ_1	58.7	0.0461	0.879
γ_2	69.1	0.028	1.085

Table – II

Calculated and experimental level energies for ^{168}Er, ^{160}Yb and ^{234}U

Bands	level	Energies (MeV) Cal.	Exp.
^{168}Er			
g	6^+	0.549	0.549
	8^+	0.932	0.928
	10^+	1.411	1.396
β_1	6^+	1.619	1.617
	8^+	1.897	1.890
	10^+	2.242	-
β_2	6^+	1.909	1.903
β_3	6^+	2.234	2.246
^{160}Yb			
g-band	6^+	1.148	1.106
	8^+	1.737	1.612
	10^+	2.374	2.144
	12^+	2.962	2.691
	14^+	3.366	3.249
	16^+	3.850	3.815
	18^+	4.478	4.387
^{234}U			
γ_1 – band	5^+	1.087	1.092
	6^+	1.159	1.172
γ_2 – band	5^+	1.275	1.275
	6^+	1.354	1.345

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