Semiclassical approach to fluctuations in temperatures of nuclei

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The excited states of a nucleus are generally closely spaced. This allows us to meaningfully evaluate statistical averages. It is well known [1] that the logarithm of density of levels $\rho(E)$ corresponds to the entropy, $S(E) = \log \rho(E)$.

The thermodynamic temperature may then be defined as $dS/dE = 1/T(E)$

$T(E)$ for nuclei can be evaluated by Bohr-Weisskopf approach [2] by representing the degrees of freedom of a nucleus as oscillations with eigenfrequencies $f_1, f_2, f_3, \ldots f_i \ldots$ We define a function $\mathcal{Z}(f)$, giving number of eigenfrequencies $f_i < f$ as a function of f. Then energy can then be written approximately as:

$$E \approx T \cdot \mathcal{Z}(T/\hbar) \qquad (1)$$

This essentially tells that the eigenfrequencies greater than $T/\hbar$ are not excited. If the excitation energies are less than the binding energy of the nucleus, then the eigenfrequencies are directly proportional to E. So, we can write $\mathcal{Z}(T/\hbar) = T/a$, where a is a constant identified as the level density parameter. Hence we get a well known formula, $E \approx T^2/a$.

This approach is basically attempting to find a "temperature-like" equivalent for quantifying quantum fluctuations. These are distinct from thermal fluctuations. For instance, thermal noise may have Gaussian distribution whereas quantum noise or fluctuations may have distribution with Gaussian core but fat-tails [3, 4], a well known example being Gumbel distribution.

We would like to relate fluctuations in temperature, as advocated above, in terms of fluctuations in level density as given by trace formula. Let us just say that trace formula provides an exact, non-perturbative approach to understand complex systems. Recently, it has been shown that 'a' can be calculated for magic and semi-magic nuclei with no adjustable parameter [7]. Further, it has been shown that the plot of a vs. A gets closer to the trend of $A/10$ with temperature.

In this paper, we try to understand that how fluctuations in the temperature vary with excitation energy. It turns out that at higher excitation energies, $\Delta T/T$ becomes independent of the excitation energies. We use the trace formula approach to calculate these fluctuations in temperature for some magic and semi-magic nuclei.

We use semiclassical trace formula for a system with spin orbit interactions [6]. The semiclassical approach shows that the single particle level density can be written as a sum of an average part $\tilde{g}$ and an oscillatory part δg.

The level density as a function of single particle level density is given by Bethe's formula.

By using the constraint relation of the mass number A, we determine ϵ_F.Using the trace formula for harmonic oscillator and spin-orbit interaction[6] and by following [7], we obtain the temperature.

The fluctuations in temperature, ΔT, about a mean T is calculated by the difference between the average temperature calculated using $\tilde{g}(E)$ and the total temperature calculated using $g(E) = \tilde{g}(E) + \delta g(E)$.

We considered ten nearly spherically symmetric nuclei namely: ^{16}O,^{40}Ca, ^{44}Ca, ^{48}Ca, ^{56}Ni, ^{90}Zr, ^{120}Sn, ^{204}Pb, ^{208}Pb, ^{210}Pb.A fluctuation measure, $\Delta T/T$ is studied with energy. It was seen that the fluctuations decrease with the increase with the energy and then finally become very slowly varying, almost independent of it.

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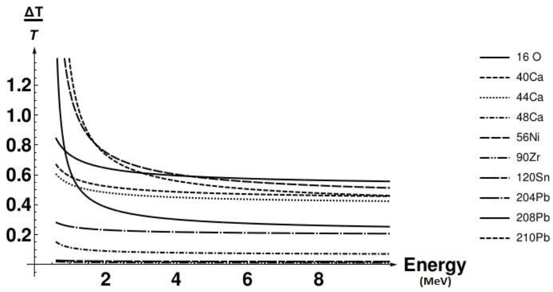

FIG. 1: Fluctuation in temperature as a function of energy for various nuclei.

However, the dependence of the fluctuations with the number of nucleons is not a simple one and one must include terms dependent on the structure of the nucleus. This opens new problems in understanding the definition of temperature its variation with mass number A. According to Feshbach [5], the dependence to be purely thermodynamic, $\Delta T/T$ varies as $1/\sqrt{A}$, where A is the number of nucleons. However, we believe that it will be more complicated than this.

Some twenty years ago, it was understood how Fermi-Dirac distribution and the Second Law [9, 10] follows on the basis of quantum chaos and random matrix theory. The average density as obtained from a Thomas-Fermi framework was thought to provide a definition of average temperature. Here we have extended and established that discussion to the level of fluctuations also.

References

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