Studying phase transition of ^{232}Th nucleons system within the BCS model

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Introduction

The Nilsson model is one of the most successful models in describing the single particle levels of nucleus. In this model the single particle energies and their spins can be deduced by the using of a modified harmonic oscillator potential. The pairing interaction between nucleons have important effect on nuclear properties and should be considered in the theoretical calculations. The pairing interaction is well described in the nuclear BCS model. The extracted results of the Nilsson model can be used as the initial data in the BCS calculations of the nuclear level density and as a result the thermal properties of the different nuclei. There are experimental data for the nuclear level densities of some nuclei. The experimental results show a constant temperature behavior at the low excitation energies [1]. This can be interpreted as an evidence for the phase transition between paired and unpaired nucleons. The BCS model is expected to describe the pairing phase transition of the nuclei. In this work we have studied the phase transition of ^{232}Th by the using of the extracted level densities and excitation energies within the BCS model and then we have compared them with the experimental results.

Models

For the number of neutrons and protons of ^{232}Th the single particle energies and their spins were first calculated using the modified harmonic oscillator potential according to the Nilsson model [2]. The oscillator quantum number $\hbar\omega_0$ has been assigned the value of $41A^{-\frac{1}{3}}$ MeV. The quantities μ and χ, which enter in the Nilsson potential, were taken from the linear relations for neutron and proton systems from the Ref [3]. The obtained results for the single particle energies and the spins have been used in the BCS calculations in order to extract the nuclear level densities of ^{232}Th. In the BCS model the most probable value for the pairing gap parameter (Δ), which is a measure of pairing interaction, and the chemical potential (λ) can be deduced as a function of temperature by solving the following equations [4]:

$$\frac{2}{G} = \sum_k \frac{1}{E_k} \tanh\frac{\beta E_k}{2} \qquad (1)$$

$$N = \sum_k (1 - \frac{\varepsilon_k - \lambda}{E_k} \tanh\frac{\beta E_k}{2}), \qquad (2)$$

where G and N are the pairing strength and the number of nucleons, respectively. Also $\beta = \frac{1}{T}$, where T is the nuclear temperature. In the Eqs.1,2 the $E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$ is the quasi particles energy and ε_k is the single particle energy. Obtained values of $\Delta(T)$ and $\lambda(T)$ can be used to compute other thermal quantities [4]:

$$E = \sum_k \varepsilon_k (1 - \frac{\varepsilon_k - \lambda}{E_k} \tanh\frac{\beta E_k}{2}) - \frac{\Delta^2}{G} \qquad (3)$$

$$S = 2\sum_k Ln(1 + \exp(-\beta E_k)) + 2\beta \sum_k \frac{E_k}{(1 + \exp(\beta E_k))}, \qquad (4)$$

where E and S are excitation energy and entropy of nucleon system, respectively.

The nuclear level density can be deduced by the following equation [3]:

$$\rho(N,Z,U)=\frac{\exp(S)}{(2\pi)^{\frac{3}{2}}D^{\frac{1}{2}}(2\pi\sigma^2)^{\frac{1}{2}}} \quad (5)$$

In the above equation, D is a determinant of the second derivatives of the grand canonical partition function taken at the saddle point. Also the σ is the spin cut off factor and can be calculated by [5]:

$$\sigma^2=\frac{1}{2}\sum_k m_k^2 \sec h^2(\frac{\beta E_k}{2}) \quad (6)$$

The experimental values of the nuclear level densities of ^{232}Th have been extracted by the Oslo group [1]. As you can see in the Fig.3 a constant temperature behavior is evident. The constant temperature behavior of the nuclear level density is a signature of phase transition. The phase transition is studied in this work.

Result and discussion

The single particle energies and their spins have been extracted by the using of the Nilsson model. Using the extracted results for the single particle energies the chemical potential and pairing gap parameter have been calculated as a function of temperature. The critical temperature of the neutron and proton system are calculated by setting $\Delta=0$ in the BCS equations. The calculated critical temperatures are T_c =0.18 and 0.3 MeV for the neutron and proton system, respectively.

The excitation energy, entropy and nuclear level densities of ^{232}Th have been extracted by the using of the Eqs. 3,4 and 5, respectively. The extracted results for the excitation energy as a function of temperature is shown in Fig.2. According to this figure the excitation energy region between 1.5 to 3.5 MeV is related to the critical temperatures and as a result the phase transition. Extracted results for the nuclear level densities are shown in the Fig.3 beside the corresponding experimental values [1]. As it can be seen in the Fig.3 the experimental nuclear level density shows a constant temperature behavior at the excitation energy region between $2\Delta\approx1.5$ to 3 MeV. The value of the constant temperature in this region is obtained through fitting the Constant Temperature Model (CTM) equation with the experimental values of the nuclear level density. Our result for this parameter is T_{CT}=0.33 MeV. This temperature is close to the critical temperature of the proton system which is obtained in the BCS model. The calculated values of the nuclear level densities in the constant temperature model are shown in Fig.3. To conclude, the extracted thermal quantities of ^{232}Th within the BCS model in this work is well describing the experimental evidence of the pairing phase transition in the nuclear level densities.

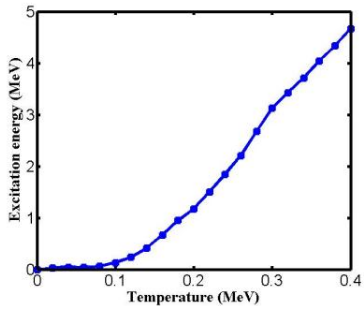

Fig.2 The extracted excitation energies for the ^{232}Th within the BCS model.

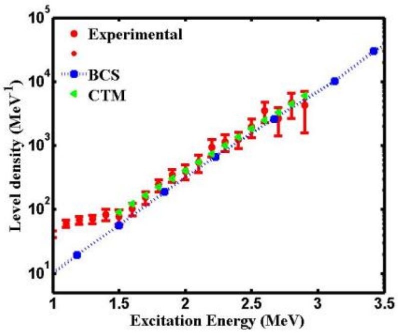

Fig.3 Extracted nuclear level densities within the BCS and CTM in comparison with the experimental values [1].

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