New perspective in the use of power law formula for ground state rotational band in transitional nuclei

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The level energies of even Z even N nuclei in the medium mass region of nuclear chart are explained by ideal rotor formula of Bohr and Mottelson [1], which contains ground state band, $K^\pi = 0^+$, β band and $K^\pi=2^+$, γ band.

$$E_I = \frac{\hbar^2 I(I+1)}{2\theta_0} \quad . \tag{1}$$

Where, I be the spin and θ_0 is moment of inertia of ground state. The energy ratio R_4 $(=(E_4/E_2))$ for spherical nuclei is 2.0 and for deformed nuclei is about 3.33 [2]. The ideal rotor formula gives the energy spectra for spherical and deformed nuclei, but adequate for shape transitional nuclei $150 \leq A \leq 200$. The reproduced energies from various energy expressions are not in agreement with experimental energy. The an-harmonic vibrator model expression of Das et al. [3] is:

$$E_I = aI + bI^2 \tag{2}$$

The soft rotor formula (SRF) of Gupta [4] and Brentano et al. [4] is;

$$E_I = \frac{\hbar^2 I(I+1)}{2\theta_0(1+\sigma I)} \tag{3}$$

The energy of ground state rotational band can also be studied by Rotation-Vibration Interaction (RVI) formula:

$$E_I = aI(I+1) + bI + cI^2(I+1) \tag{4}$$

The level energy can also be obtained by variable moment of inertia (VMI) model of Mariscotti et al. [5] as:

$$E_I = \frac{I(I+1)}{2\theta_I} + \frac{C(\theta_I-\theta_0)^2}{2} \quad . \tag{5}$$

Where, moment of inertia θ_I is determined for each spin I. The C is a restoring force constant and θ_0 is ground state moment of inertia. Gupta et al. [2] proposed a single term energy formula termed as power law (PL) and the energy is expressed as:

$$E_I = aI^b \tag{6}$$

It was used by Mittal et al. [6] to study the light mass region Xe- Gd. The power law was used [7, 8, 9] to study the Mallmann's plot, $K^\pi=0^+$ β and for low mass region $20 \leq A \leq 120$.

Here the level energies are reproduced using Power law and Soft Rotor Formula, using level energies for spin $I \leq 12$ below back bending and compared with experimental level energies [10]. It is observed that the root mean square deviation RMSD (D) is small in Power Law (see Table 1), whereas in Soft Rotor Formula it is large. The RMSD (D) is written as:

$$D = \sqrt{\sum_{i=1}^{n} \frac{1}{(n-m)} (E_{Expt} - E_{Cal})^2} \tag{7}$$

Result

The power law suggests that the spin is varied with non integer spin more appropriately than other energy expressions. The PL gives the better fit of the energy because the RMSD is very smaller in comparison to SRF for Nd nuclei.

Table 1. The experimental [10] and theoretical values of energy from power law (PL) and soft rotor formula (SRF) for ground band for $^{146-156}$Nd.

	A	E_2	E_4	E_6	E_8	E_{10}	E_{12}	RMSD
Exp	^{146}Nd	453.8	1042.2	1780.0	2474.5	3123.8	3902.0	
PL		453.3	1051.1	1719.1	2437.2	3195.0	3986.0	65.34585
SRF		453.8	1042.2	1669.4	2313.1	3234.9	4205.3	142.4035
Exp	^{148}Nd	301.7	752.3	1279.8	1856.2	2471.2	3106.2	
PL		301.1	746.9	1270.6	1852.4	2481.5	3151.2	36.313
SRF		301.7	752.3	1261.9	1800.9	2570.8	3342.0	262.1393
Exp	^{150}Nd	130.2	381.1	720.2	1129.6	1598.5	2118.7	
PL		130.3	383.2	720.1	1126.6	1594.3	2117.2	6.701017
SRF		130.2	381.1	713.4	1103.1	1674.3	2176.6	100.4109
Exp	^{152}Nd	72.4	236.5	484.0	806.2	1196.2	1648.7	
PL		72.7	241.6	487.8	803.0	1181.9	1621.0	36.00131
SRF		72.4	236.5	486.9	818.6	1338.6	1740.1	249.1005
Exp	^{154}Nd	70.8	233.2	481.9	810.1	1210.8	1677.3	
PL		71.2	239.6	487.3	806.4	1191.8	1640.0	47.81608
SRF		70.8	233.2	484.0	820.1	1351.1	1756.4	231.491
Exp	^{156}Nd	67.2	222.2	460.7	778.2	1169.0	1628.4	
PL		67.6	229.0	467.4	775.4	1148.3	1582.7	55.17385
SRF		67.2	222.2	462.9	787.3	1301.8	1692.4	208.0467

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