Study of ground, γ and γγ – bands in ^{112}Ru nucleus

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In past few decades, extremely rich experimental data have been accumulated in low – lying nuclear spectroscopy. In context of ^{112}Ru nucleus, Sakai [1] in 1984 illustrated few energy levels for ground state band ($K^{\pi} = 0^{+}$) and none for other bands. Recently S. Lalkovski et al [2] compiled the experimental data for A = 112 nucleus for different bands upto higher energy levels. In neutron rich nuclei A ≥ 100 the nuclear shape changes rapidly as the valance nucleons fill the $g_{9/2}$ proton and $h_{11/2}$ neutron orbitals. The researchers also expect that the presence of two opposite shape driving unpaired nucleons from high j – orbital could stabilize the γ – soft core into a rigid triaxial shape [3]. The ramifications on the nuclear structure because of various shapes make these neutron rich nuclei an ideal testing ground for various theoretical approaches [4].

The basic results of the fundamental models of Davydov – Filippov [5] and Wilets – Jean [6] connecting the band head energies are of immense significance since no perturbation takes place in them because of any physical effects. According to these models, $E3_1^+ = E2_1^+ + E2_2^+$ for γ – rigid nucleus and $E3_1^+ = 2E2_1^+ + E4_1^+$ for γ – soft nucleus. Thus, we can take $\Delta E_1 = E3_1^+ - (E2_1^+ + E2_2^+) \approx 0$ for γ – rigid and $\Delta E_2 = E3_1^+ - (2E2_1^+ + E4_1^+) \approx 0$ for γ – soft nucleus. For ^{112}Ru, $\Delta E_1 = 13\ KeV$ and $\Delta E_2 = 371\ KeV$ which reflects the nucleus as γ – rigid. The ratio $R\left(\frac{4}{2}\right) = \frac{E4_1^+}{E2_1^+}$ is 2.71 and $\beta A^{2/3}$ = 7.0 which primarily show the deformed shape. Earlier this nucleus has been commented axial rotor at low spin [7].

Another signature of triaxiality is odd – even staggering (OES) in the γ – band. The staggering indices S (J) is calculated using relation $S(J) = \frac{(E4_2^+ - E3_1^+) - (E3_1^+ - E2_2^+)}{E2_1^+}$ and is plotted with spin as shown in fig. 1. Initially S (J) is very small and one sided which reflects the axial nature at low spin regime but, as spin increases beyond 7_1^+, the value of S (J) increases and nucleus becomes triaxial. This expected triaxiality motivates to explore the structure of ^{112}Ru in the frame work of rigid triaxial rotor model (RTRM).

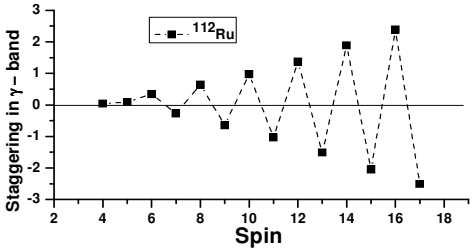

Fig. 1

In the present study we have employed RTRM of Davydov and Filippov to study the excitation energies of ground band (i.e. $K^{\pi} = 0^{+}$) and anomalous rotational band (i.e. $K^{\pi} = 2^{+}$) known as the γ – band in the literature. We also includes the study of a newly known band (i.e.$K^{\pi} = 4^{+}$) γγ – band on the band head energy of $4_3^+ = 1413\ KeV$. The asymmetry parameter (γ) for ^{112}Ru nucleus is evaluated from the two band head energy ratio $\frac{E2_2^+}{E2_1^+}$ using the relation –

$$\frac{E2_2^+}{E2_1^+} = \frac{1+\left[1-\frac{8}{9}\sin^2 3\gamma\right]^{1/2}}{1-\left[1-\frac{8}{9}\sin^2 3\gamma\right]^{1/2}}$$

The asymmetry parameter comes out to be 26.3^0 which is used to evaluate the RTRM energies of different bands under consideration. The RTRM energies have been normalized using Lipas like relation -

$$E(fit) = \frac{E_{RTRM}}{1+\alpha E_{RTRM}}$$

Where, α is Lipas parameter. The Lipas like relation is used earlier by several authors [7- 9]. For ^{112}Ru nucleus, we have taken

different values of Lipas parameter for ground, even spin and odd spin branch of γ – band and also for γγ – band. The parameters used in the present work are listed in table – I.

Table – I

List of parameters used in present work for ^{112}Ru.

γ	g. s. band	γ - band		γγ – band
	α	α_{even}	α_{odd}	α
26.3^0	4.5×10^{-5}	2.0×10^{-4}	1.0×10^{-4}	1.75×10^{-4}

The values of RTRM and experimental energies for ^{112}Ru nucleus in ground, γ and γγ – bands are listed in table – II, III & IV respectively. The energies calculated for all three bands are compared with experimental values which are in fine agreement.

Table – II

Ground state band energies in KeV

Spin	Exp.	E(fit)
2^+	236.6	234.1
4^+	644.9	635.8
6^+	1189.8	1165.0
8^+	1839.7	1807.6
10^+	2563.0	2541.7
12^+	3362.2	3343.6
14^+	4118.4	4191.3
16^+	4954.6	5064.5
18^+	5830.0	5945.3
20^+	6725.4	6819.3

Table – III

γ – band ($K^\pi = 2^+$) energies in KeV

Spin/Energy	Exp.	E(fit)
2^+	523	476.0
3^+	747.5	708.6
4^+	980.7	1031.9
5^+	1235	1283.5
6^+	1570.2	1627.2
7^+	1841.1	1918.4
8^+	2263.5	2116.2
9^+	2534.2	2560.5

Table – IV

γγ – band ($K^\pi = 4^+$) energies in KeV

Spin	Exp.	E(fit)
4^+	1413	1402.1
5^+	1649	1660.2
6^+	1955	1968.9
7^+	2231	2204.8
8^+	2574	2575.0
9^+	2909	2701.5

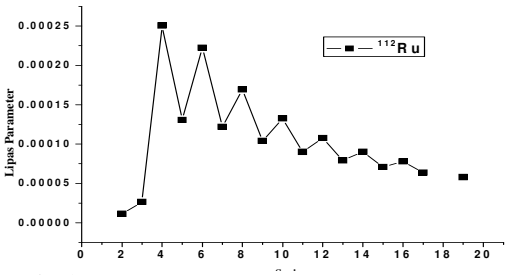

Fig. 2 Variation of Lipas parameter for γ_1-band in ^{112}Ru nucleus

Conclusion:

In the present study, RTRM has been employed in which the projection of angular momentum along $\hat{1}$-axis causing rotational band and another projection of angular momentum in ($\hat{2}$, $\hat{3}$) plane that produce anomalous rotational bands. The employment of RTRM with Lipas parameter describes the energies of the different bands

It is inferred that the nucleus is axial rotor at low spin up to 6_2^+ and it becomes triaxial beyond that spin. It is further commented that if the Lipas parameter are considered variable and are calculated at different spins then we note that the values of parameter α become almost same at spin beyond 14_2^+ (fig.2). This is an indicator that the nucleus is more and more rigid (lesser value of α) at higher spins.

Acknowledgement:

Authors acknowledged the Chairman, Dr. O. P. Agrawal and the Director, Prof. Ajay Kumar, NIET, Gr. Noida for providing research facilities.

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