Bound state wave function for deuteron stripping reaction on ^{16}O by Gaussian quadrature method

A. Acharya,* R. L. Nayak & T. Sahoo
Department of Physics, Veer Surendra Sai University of Technology, Odisha-768018, INDIA
*email: aacharya_phy@vssut.ac.in

The two-body interaction which is used as an input, separability of t-matrix introduces considerable simplification in the sense that it reduces the dimensionality of the coupled integral equation from two to one in order to save the computing time. Hence the cross-section can be easily measured accurately for higher order square matrix.
When the two body potential is separable i.e.

$< k' | V | k> = -\lambda g(k')g(k)$ (1)

Then the two-body t-matrix is also separable [1]

$t(z) = V_k | \varphi_k^{n_k} > \tau_k^{n_k} < \varphi_k^{n_k} | V_k$ (2)

Where $\tau_k^{n_k}$ is the two body t-matrix propagator

$\tau_k^{n_k} = \frac{1}{\lambda'} - < \varphi_k^{n_k} | V_k g_o(z) V_k | \varphi_k^{n_k} >$ (3)

$\varphi_k^{n_k}$ is the bound state wave function and $g_o(z)$ is the free resolvant operator

$\frac{1}{\lambda'} = < \varphi_k^{n_k} | V_k g_o(-\epsilon_{B_k}^{n_k}) V_k | \varphi_k^{n_k} >$ (4)

A justification for the separability is given by Lovelace [2] and then by Fuda. Based on their approximations the Alt Grassberger and Sandhas (AGS) version of Faddeev approach become more suitable to solve few body problems [3].
The one dimensional coupled integral equation is written as

$T_{ij}\,(q_i, q_j', \beta_i, \beta_j : J) = k_{ij}(q_i, q_j', \beta_i, \beta_j : J) + \sum_{k\beta_k} \int u_k^2 du_k K_{ik}(q_i, u_k, \beta_i, \beta_k : J) \tau_k^{n_k}(z - u_k^2) T_{kj}(u_k, q_j', \beta_k, \beta_j : J)$ (5)

The transformation co-efficient (TC) appearing in the coupled integral equation can be easily evaluated by applying graphical method of spin algebra [4].The mod square of the transformation co-efficient gives the probability to find the final product after certain rearrangement in the initial channel.

$$TC = \sum_L \iint dq_i du_j F_L(q_i, u_j)$$

$$(\epsilon_{ij})^{s_i+s_j-2s_k-s_j-s_i} \frac{1}{p_i^{L_j}} \frac{\sqrt{4\pi}}{(\lambda_j)} \left(\frac{1}{B_{ij}}\right)^{L_j+3} (\epsilon_{ij})^{s_i+s_j-2s_k-s_j}$$

$$F_L(q_i, u_i) \sum_{\lambda_j=0}^{L_i} (A_{ij})^{\lambda_j} u_j^{\lambda_j} q_i^{L_j+\lambda_j} \epsilon_{ij}^{L_i} \frac{1}{[j]^2} XYZ \quad (6)$$

Where

$$X = \frac{[L][L_j-\lambda_j][l_i][L][L_j][\lambda_j][l_j]}{4\pi} (-)^{2L+2L_j} \begin{pmatrix} L & l_j-\lambda_j & l_i \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & l_j & \lambda_j \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_i & L_j-\lambda_j & L \\ \lambda_j & l_j & L_j \end{pmatrix} \quad 7(a)$$

$$Y = (-)^{2S_i} \begin{bmatrix} l_i & L_j & l_j \\ k_j & J & s_i \end{bmatrix} [k_j][J] \quad 7(b)$$

And

$$Z = [k_j][J_j][S_i](-)^{2L_j}(-)^{2s_k}(-)^{2s_i}(-)^{2J_j} \begin{bmatrix} S_i & s_k & s_j \\ J_j & k_j & L_j \end{bmatrix} \quad 7(c)$$

By considering a suitable angular momentum basis, the coupled AGS equation can be written in one dimensional form. The one dimensional coupled AGS equation then also contains logarithmic and pole singularities. By suitable methods devised by Sasakawa and Kowalski [5, 6] these singularities can be overcome.

By taking 15-point Gaussian quadrature for special weights and momentum points the integral from 0 to ∞ can be divided into three different limits such as (i) from 0 to $\sqrt{E}$ (ii) $\sqrt{E}$ to $\sqrt{(E+2E_B)}$ and (iii) $\sqrt{(E+2E_B)}$ to ∞

Then the two-body bound state wave function can be related with the form factor as [7]

$$\varphi_n(\mathrm{p}) = \frac{N_{lj}g_{lj}(p)}{(p^2+\alpha^2)} \quad (8)$$

$$\frac{1}{N_{lj}^2} = \int \frac{g_{lj}(p)}{(p^2+\alpha^2)}\, d^3p \quad (9)$$

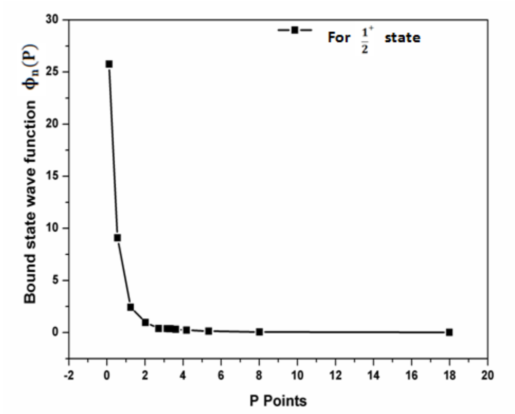

Figure 1: Momentum points vs bound state wave function for $^{16}O(d,n)^{17}F$ in $S_{1/2}$ state

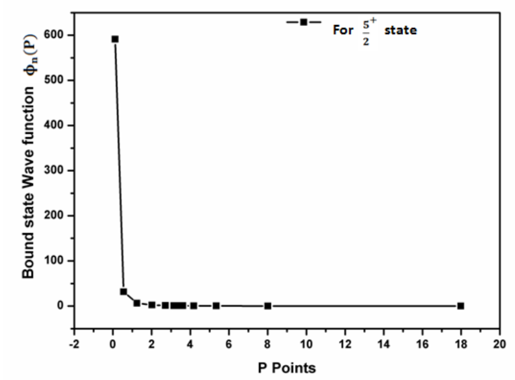

Figure 2: Momentum points vs. bound state wave function for ^{16}O (d, p) ^{17}O in $d_{5/2}$ state

Result and discussions:

The AGS equation is written in solvable one dimensional coupled integral equation by choosing a suitable angular momentum basis. Taking the separable approximation of two-body t-matrix in AGS equation and by considering both short and long range interaction part the wave function can be evaluated. The bound state wave function versus momentum points graph for ^{17}F ($S_{1/2}$ state) and ^{17}O ($d_{5/2}$ state) is shown in fig. 1 and fig. 2 respectively. From both the figure it is observed that bound state wave function decreases with increase in momentum even the coulomb interaction may or may not be considered. Our future plan is to find out the variation in bound state wave function with momentum by considering the allowed states of higher elements in stripping reaction case.

References:

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