Pseudoscalar decay constant of B and B_s mesons using Dirac formalism

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Introduction

The pseudoscalar decay constants are of great interest since they enter as input in non-leptonic B_q decays, in the hadronic matrix elements of $B_q - \bar{B}_q$ mixing, and in the extraction of CKM matrix elements $|V_{cb}|$, $|V_{ub}|$ from the leptonic decay widths of B mesons. In the ongoing quest for new effects in high-energy particle physics, flavour physics provides information complementary to that from the direct searches performed at ATLAS and CMS. There is no direct evidence for decay constant from experimental side. Recently, two approaches, QCD sum rules (QCDSR) [1–4] and lattice QCD (LQCD) [5, 6] found the pseudoscalar decay constant of B and B_s mesons.

Theoretical Framework

The decay constant (f_p) of a pseudoscalar meson B_q consisting of a heavy b-quark and a lighter q-quark, with q = u, d, s, is defined through the matrix element of the pseudoscalar current [7]

$$\langle 0|\bar{q}\gamma^{\mu}\gamma_5 c|P_{\mu}\rangle = if_p\ P^{\mu} \tag{1}$$

In the relativistic quark model, the decay constant can be expressed through the meson wave function in the momentum space [8, 9]

$$f_P = \left(\frac{3|I_p|^2}{2\pi^2 M_p\ J_p}\right)^{\frac{1}{2}} \tag{2}$$

Here M_p is mass of the pseudoscalar meson and I_p and J_p are defined as

$$I_p = \int_0^{\infty} dp\ p^2 A(p)[G_{q1}(p)G^*_{q2}(-p)]^{\frac{1}{2}} \tag{3}$$

$$J_p = \int_0^{\infty} dp\ p^2 [G_{q1}(p)G^*_{q2}(-p)] \tag{4}$$

respectively. Where,

$$A(p) = \frac{(E_{p1} + m_{q1})(E_{p2} + m_{q2}) - p^2}{[E_{p1}\ E_{p2}(E_{p1} + m_{q1})(E_{p2} + m_{q2})]^{\frac{1}{2}}} \tag{5}$$

and $E_{p_i} = \sqrt{{k_i}^2 + m^2_{q_i}}$.

The potential model used here to obtain the constituent quark orbital corresponding to the ground state of the mesonic system assumes the quark and antiquark inside the meson to be independently bound in an average flavor-independent phenomenological potential of the form [10, 11]

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda r^{\nu} + V_0) \tag{6}$$

The independent quark Lagrangian density in zeroth order is given as

$$\mathcal{L}^0_q(x) = \bar{\Phi}_q(x)\left[\frac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial_{\mu}} - V(r) - m_q\right]\Phi_q(x). \tag{7}$$

The normalized quark wave functions $\Phi(\vec{r})$ obtained from eqn. (7) satisfies the Dirac equation given by

$$[\gamma^0 E_q - \vec{\gamma}.\vec{P} - m_q - V(r)]\Phi_q(\vec{r}) = 0. \tag{8}$$

The two component solution of Dirac equation can be written as

$$\Phi_{nlj}(r) = \begin{pmatrix} \Phi_A \\ \Phi_B \end{pmatrix} \tag{9}$$

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TABLE I: Pseudoscalar decay constant (f_P) of B system (in MeV).

	f_P			
	1S	2S	3S	4S
Present	188.56	328.13	440.88	533.35
$[QCDSR]$ [1]	186 ± 14			
$[CPP_\nu]$ [13]	192			
$[QCDSR]$ [2]	206 ± 7			
$[RPM]$ [14]	198 ± 14			
$[LFQM]$ [15]	204.0 ± 31			
$[QCDSR]$ [3]	190 ± 17			
$[QCDSR]$ [4]	$207\ ^{+17}_{-9}$			
$[LQCD]$ [5]	219 ± 17			
$[LQCD]$ [6]	196.2 ± 15.7			
$[LQCD]$ [12]	196.9 ± 9.1			
$QAPM$ [16]	216			

TABLE II: Pseudoscalar decay constant (f_P) of B_s system (in MeV).

	f_P			
	1S	2S	3S	4S
Present	240.21	393.61	521.26	614.28
$[QCDSR]$ [1]	222 ± 12			
$[CPP_\nu]$ [13]	217			
$[QCDSR]$ [2]	234 ± 5			
$[RPM]$ [14]	237 ± 17			
$[LFQM]$ [15]	270.0 ± 47			
$[QCDSR]$ [3]	233 ± 17			
$[QCDSR]$ [4]	$242.0\ ^{+17}_{-12}$			
$[LQCD]$ [5]	264 ± 19			
$[LQCD]$ [6]	235.4 ± 12.2			
$[LQCD]$ [12]	242.0 ± 10.0			
$QAPM$ [16]	232			

where the positive and negative energy solutions are written as

$$\Phi_A^{(+)}(\vec{r}) = N_{nlj}\begin{pmatrix}\frac{ig(r)}{r}\\ \frac{(\sigma.\hat{r})f(r)}{r}\end{pmatrix}\mathcal{Y}_{ljm}(\hat{r}) \quad (10)$$

$$\Phi_B^{(-)}(\vec{r}) = N_{nlj}\begin{pmatrix}\frac{i(\sigma.\hat{r})f(r)}{r}\\ \frac{g(r)}{r}\end{pmatrix}(-1)^{j+m_j-l}\mathcal{Y}_{ljm}(\hat{r}) \quad (11)$$

The radial solutions $f(r)$ and $g(r)$ is obtained numerically to yield the energy eigen values. The parameters are fixed to get the ground state masses of B and B_s mesons. The quark mass parameters m_b, $m_{u,d}$ and m_s are taken as 4.67 GeV, 0.003 GeV and 0.1 GeV respectively.

The computed decay constant (f_{B_q}) are listed in Table I and Table II. We have compared our results with the latest predictions by QCDSR, LQCD and other theoretical results.

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