Molecular-like states in the bottomonium sector

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Introduction

The molecular interpretation of the bottomonium like states have been proposed and studied since long time [1–5]. Recently, the BELLE collaboration reported the charge narrow states $Z_b(10650)$ and $Z_b(10610)$ in the $\Upsilon(5S)$ decay, which do not fit in $b\bar{b}$ scheme [2, 3]. The molecular interpretation of the states have been proposed by authors in various approaches like local hidden gauge approach [3], potential model [4, 5], QCD sum rule [6]. Here, we studied the dimesonic states which is assumed to be as a bound state of meson-antimeson, just like deuteron, in the potential model framework. We have used the Yuwka like potential with One Pion Exchange Potential for the mass calculation of the dimesonic systems.

Theoretical framework

We have solved the Schrödinger equation by using the virial theorem and hydrogen like trial wave function in the Ritz variational scheme. The Hamiltonian of the meson-antimeson system is given by the equation [7–9]

$$H = \sqrt{P^2 + m_a^2} + \sqrt{P^2 + m_b^2} + V(r_{12}) \quad (1)$$

where $V(r_{12})$ is the interaction potential, consists of the Yukawa like potential and one pion exchange potential (OPEP)

$$V(r_{12}) = \frac{\alpha_s^2}{r_{12}} e^{\frac{(Cr_{12})^2}{2}} + V_\pi \quad (2)$$

Where α_s is residual running coupling constant [7, 8] and C (=0.2) is the screening parameter. The V_π is one pion exchange potential, which takes the form namely [10, 11],

$$V_\pi = \frac{1}{3}\frac{g^2{}_8}{4\pi}\left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2}\right)(\tau_i \cdot \tau_j)(\sigma_i \cdot \sigma_j) \left(\frac{e^{-m_\pi r_{ij}}}{r_{ij}} - \left(\frac{\Lambda_\pi}{m_\pi}\right)^2 \frac{e^{-\Lambda_\pi r_{ij}}}{r_{ij}}\right) \quad (3)$$

where τ and σ are the usual isospin and spin matrices and Λ_π (=0.029) is the form factor, g_8 (=0.69) is the pion-meson coupling constant [11]. The spin dependent potential has been incorporated perturbatively, as in Ref. [7, 8]. By using the hydrogen like trial wave function and virial theorem, we have extracted the masses of dimesonic states [7–9]

$$\langle K.E. \rangle = \frac{1}{2}\left\langle \frac{rdV(r)}{dr} \right\rangle \quad (4)$$

Results and Discussion

The masses of the molecular dimesonic states with their possible $I^G(J^{PC})$ values and root mean squared radius are tabulated in the Table - I. Our results are in good agreement with experimental [1, 2] as well as other theoretical studies [3–6]. We have found few dimesonic states in close mass spectra, which can be compared with some promising molecular states, like $Z_b(10610)$ and $Z_b(10650)$. We compared the $Z_b(10610)$ with $B-\overline{B^*}$(J^{PC}-1^+ (1^{+-})) while $Z_b(10650)$ with $B^*-\overline{B^*}$ with I=1 and possible J=0 or 2. Some other states are also compared as found in literature, shown in Table - I. For confirmation of the molecular interpretation of these states, more study is needed with respective to their decay and coupling which is the future scope of the study.

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TABLE I: Mass spectra of dimesonic ($q\overline{Q} - \overline{q}Q$) systems with their J^{PC} values

System	$I^G(J^{PC})$	μ (GeV)	R(0) ($GeV^{\frac{3}{2}}$)	$\sqrt{< r^2 >}$ (fm)	B.E. (MeV)	Mass (GeV)	others (GeV)	others (GeV)	State
$B-\overline{B}$	0^+ (0^{++})	0.958	0.663	0.71	-43.06	10.516	11.100 [5]		
$B-\overline{B_s}$	0^+ (0^{++})	1.026	0.735	0.66	-51.44	10.594			
$B_s-\overline{B_s}$	0^+ (0^{++})	0.967	0.672	0.70	-43.46	10.690			
$B-\overline{B_c}$	0^+ (0^{++})	1.047	0.757	0.65	-51.58	11.505			
$B_s-\overline{B_c}$	0^+ (0^{++})	1.055	0.766	0.64	-52.02	11.591			
$B_c-\overline{B_c}$	0^+ (0^{++})	1.061	0.773	0.64	-47.43	12.506			
$B-\overline{B^*}$	0^- (1^{+-})	1.111	0.828	0.61	-62.54	10.542			
$B-\overline{B^*}$	1^+ (1^{+-})	0.993	0.700	0.68	-47.46	10.557	10.607 [2]	10.587 [3]	$Z_b(10610)$
$B_s-\overline{B_s^*}$	0^- (1^{+-})	1.058	0.769	0.64	-54.27	10.727	11.373 [5]		
$B_c-\overline{B_s^*}$	0^- (1^{+-})	1.209	0.940	0.56	-68.72	11.623			
$B^*-\overline{B^*}$	0^+ (0^{++})	1.279	1.022	0.53	-88.68	10.542	10.582 [4]		
$B^*-\overline{B^*}$	0^- (1^{+-})	1.195	0.923	0.57	-74.84	10.567			
$B^*-\overline{B^*}$	0^+ (2^{++})	1.018	0.727	0.67	-50.76	10.604	10.602 [4]		
$B^*-\overline{B^*}$	1^- (0^{++})	1.047	0.758	0.65	-54.45	10.585	10.652 [2]	10.650 [3]	$Z_b(10650)$
$B^*-\overline{B^*}$	1^+ (1^{+-})	1.077	0.790	0.63	-58.27	10.586			
$B^*-\overline{B^*}$	1^- (2^{++})	1.136	0.856	0.60	-66.29	10.590	10.652 [2]	10.650 [3]	$Z_b(10650)$
$B_s^*-\overline{B_s^*}$	0^+ (0^{++})	1.154	0.876	0.59	-66.59	10.752			
$B_s^*-\overline{B_s^*}$	0^- (1^{+-})	1.062	0.774	0.64	-54.53	10.771			
$B_s^*-\overline{B_s^*}$	0^+ (2^{++})	0.883	0.587	0.77	-34.02	10.799	11.386 [5]		
$B_s^*-\overline{B_s^*}$	1^- (0^{++})	0.913	0.617	0.74	-37.11	10.787			
$B_s^*-\overline{B_s^*}$	1^+ (1^{+-})	0.942	0.647	0.72	-40.33	10.787			
$B_s^*-\overline{B_s^*}$	1^- (2^{++})	1.002	0.709	0.68	-47.17	10.787			

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