Isospin in $NN \to NN\pi$

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Introduction

Pion production in NN collisions is not only sensitive to short range NN interaction, but includes also NN transitions between singlet and triplet spin states, which are forbidden in elastic scattering. At threshold energies, participation of only a few partial waves in the final state is involved, which in turn limit the initial partial waves through Parity conservation and Pauli principle.

Ever since the total cross section measurements [1] for $pp \to pp\pi^0$ were found to be more than a factor of 5 larger than the then available theoretical predictions, experimental and theoretical study of the reaction excited considerable interest. Advances in storage ring technology led to detailed experimental studies including measurements [2] when both the colliding protons are polarised. Theoretical models including the more advanced Julich meson exchange model [3] failed to provide an overall satisfactory reproduction of the data.

A model independent approach developed earlier [4] was employed [5] to analyse the findings of the Julich model. This study identified not only some of the short comings of the model, but also revealed that the Δ contributions are important. It was pointed out subsequently [6] that there was a phase ambiguity in [5] and it was shown how this drawback can be removed through additional measurements of spin observables.

The purpose of this contribution is to present a simple isospin analysis of the $NN \to NN\pi$ and identify also the partial waves in those channels, which allow Δ contributions.

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TABLE I: List of partial wave amplitudes for $NN \to NN\pi$ with $I_i = 1$ and $I_f = 0$.

Initial NN state	Type	Final $NN\pi$ state	$M^{I_f I_i;j}_{l(l_f s_f)j_f;l_i s_i}$
$^3P_1$	Ss	$^3S_1, s$	$M^{01;1}_{0(01)1;11} = f^{01}_1$
$^1S_0$	Sp	$^3S_1, p$	$M^{01;0}_{1(01)1;00} = f^{01}_2$
$^1D_2$		$^3S_1, p$	$M^{01;2}_{1(01)1;20} = f^{01}_3$
$^3P_0$	Pp	$^1P_1, p$	$M^{01;0}_{1(10)1;11} = f^{01}_4$
$^3P_1$		$^1P_1, p$	$M^{01;1}_{1(10)1;11} = f^{01}_5$
$^3P_2$		$^1P_1, p$	$M^{01;2}_{1(10)1;11} = f^{01}_6$
$^3F_3$		$^1P_1, p$	$M^{01;2}_{1(10)1;31} = f^{01}_7$

Isospin Analysis

In the particular case of $pp \to pp\pi^0$, the two nucleon system makes a transition from an initial state with isospin I_i=1 to a final state with isospin I_f=1, whereas in $pp \to pn\pi^+$, the initial state of the NN system is I_i=1, but the final state is a linear combination of I_f= 0,1. In $pn \to pp\pi^-$, the initial state is a linear combination of I_i= 0,1 while the final state has I_f=1. In $pn \to pn\pi^0$, the NN system in the initial as well as in the final states involves linear combinations of isosinglet and isotriplet states. Since a pion with isospin $I_\pi = 1$ is produced in the final state, isospin conservation in the reaction demands that the reaction amplitudes must be related through Clebsch-Gordan coefficients $C(I_f, 1, I_i; m_f, m_\pi, m_i)$, if m_f,m_π,m_i denote the respective eigen values of the z-components of isospin. Thus the reaction matrices M in spin space for a) $pp \to pp\pi^0$, b) $pp \to pn\pi^+$, c) $pn \to pp\pi^-$, d) $pn \to pn\pi^0$ and e)$pn \to nn\pi^+$ are given respectively by

$$M = M^{1,1}/\sqrt{2} \quad (1)$$

$$M = (\sqrt{2}M^{0,1} - M^{1,1})/2 \quad (2)$$

$$M = (\sqrt{3}M^{1,1} + \sqrt{2}M^{1,0})/2\sqrt{3} \quad (3)$$

$$M = (\sqrt{3}M^{0,1} + M^{1,0})/2\sqrt{3} \quad (4)$$

$$M = (\sqrt{3}M^{1,0} - \sqrt{2}M^{1,1})/\sqrt{6} \quad (5)$$

Each of the M^{I_f,I_i} may be expressed in the NN spin space in the form [4, 6]

$$M^{I_f,I_i} = \sum_{s_f,s_i=0}^{1} \sum_{\lambda=|s_f-s_i|}^{|s_f+s_i|} (S^\lambda(s_f,s_i).M^\lambda), \quad (6)$$

where the irreducible tensor amplitudes $M^\lambda_\mu = M^\lambda_\mu(I_f s_f; I_i s_i)$, of rank λ are given in terms of partial wave reaction amplitudes $M^{I_f I_i;j}_{l(l_f s_f)j_f;l_i s_i}$. These are dependent on c.m energy E and the invariant mass W of the final NN system. In the particular case of $pp \to pp\pi^0$, it clear from (1) that only the $I_f = I_i = 1$ partial waves are involved. It should also be noted that Δ contributions [7] can arise in $M^{1,1}$ and $M^{0,1}$ and not in $M^{1,0}$, since Δ with isospin 3/2 and N with isospin 1/2 cannot combine to give the conserved isospin $I_i = 0$. Focusing attention on the Δ contributions [7], the partial wave reaction amplitudes with $I_i = 1$ and $I_f = 0$ have been worked out with l_f, l going up to $l_f = 1$ and $l = 1$. The $M^{I_f I_i;j}_{l(l_f s_f)j_f;l_i s_i}$ for$I_f = 0, I_i = 1$ are shown in TABLE I. It is clear from (1) that the $M^{j}_{l(l_f s_f)j_f;l_i s_i}$ listed in the Table I of [6] are $M^{I_f I_i;j}_{l(l_f s_f)j_f;l_i s_i}$ with $I_f = I_i = 1$. There are 12 allowed amplitudes with $I_f = I_i = 1$, whereas there are only seven allowed amplitudes with $I_f = 0, I_i = 1$, as shown in TABLE I here. In general, one has to work out also the allowed $M^{I_f I_i;j}_{l(l_f s_f)j_f;l_i s_i}$ for $I_f = 1, I_i = 0$ and all the three have to be used together following Eqs. (1) to (6) to discuss the $NN \to NN\pi$ reactions and assess the importance of Δ contributions. Moreover, the identification of the spin observables in b) $pp \to pn\pi^+$, c) $pn \to pp\pi^-$, d) $pn \to pn\pi^0$ and e)$pn \to nn\pi^+$ becomes more complicated than in the case of a) $pp \to pp\pi^0$ to empirically determine the reaction amplitudes as in $pp \to pp\omega$ [8]. Further work is in progress.

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