Spectral Function of Δ and π-N Cross Section in hot and dense medium

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The π-N cross section is known to be dominated by the Δ resonance in the region of centre of mass energy peaked around $m_\Delta = 1232$ MeV. Consequently the behaviour of the Δ at finite temperature and baryon density can be used to determine the $\pi - N$ cross section in the medium in this energy region. Here we evaluate the self-energy of the Δ using the real time formulation of thermal field theory to obtain its spectral function. We then calculate the π-N cross-section using a phenomenological model introducing the width in the propagator of the exchanged Δ which reproduces the measured cross-section fairly well. The in-medium cross-section obtained by replacing the vacuum width by the thermal one shows a significant modification. Among others, this can have observable effects on the magnitude of transport coefficients in hot/dense matter produced in relativistic heavy ion collisions.

The spectral function of the Δ is given by the imaginary part of the exact propagator Σ, which in turn is obtained by solving the Dyson-Schwinger equation

$$\Sigma = \Sigma^0 + \Sigma^0 \Pi \Sigma \qquad (1)$$

where Σ^0 is the vacuum propagator and Π is the self-energy. We get

$$\Sigma(q) = \frac{-1}{q^2 - m_\Delta^2 + \Pi} \qquad (2)$$

where

$$\Pi = \sum_{s_\Delta} \bar{\Psi}^\mu \Pi_{\mu\nu} \Psi^\nu \qquad (3)$$

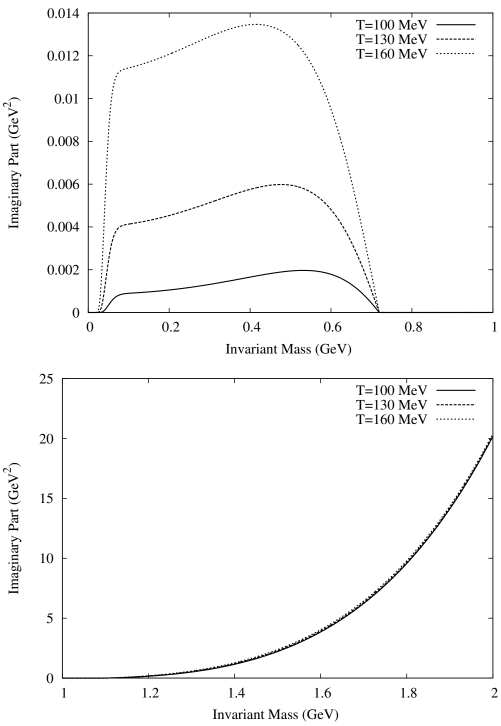

FIG. 1: The Landau and unitary-cut contribution in the imaginary part of the Δ self -energy.

with Ψ^μ and $\Pi_{\mu\nu}$ denoting the spin 3/2 spinor and self-energy of the Δ respectively.

Consequently, the spectral function is given by

$$\text{Im}\Sigma(q) = \frac{\text{Im}\Pi}{(q^2 - m_\Delta^2 + \text{Re}\Pi)^2 + (\text{Im}\Pi)^2} \qquad (4)$$

Using the Lagrangian

$$\mathcal{L} = \frac{g_\Delta}{\sqrt{2}} \bar{\psi}_a (\partial^\mu \vec{\pi} \cdot \vec{\tau})^c_b \Psi^{abd}_\mu \epsilon_{cd} + h.c. \qquad (5)$$

we evaluate the $\pi - N$ loop at finite temperature and baryon density to obtain the real

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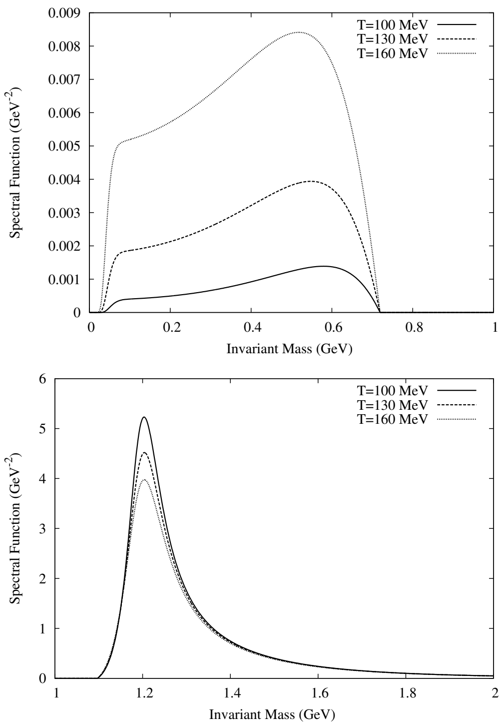

FIG. 2: The Landau and unitary-cut contribution of the spectral function of Δ.

and imaginary parts of Π. The Δ and nucleon fields are denoted by Ψ_μ and ψ respectively and the indices a, b, c, d take values 1 & 2 with $\epsilon_{12} = -\epsilon_{21} = 1$. We take $g_\Delta = 18.4$ GeV^{-1}.

The vacuum self-energy is given by

$$\Pi_{\mu\nu}(q) = ig_\Delta^2 \int \frac{d^4k}{(2\pi)^4} k_\mu k_\nu \Delta(k) S(q-k) \quad (6)$$

where $\Delta(k) = \frac{-1}{k^2-m_\pi^2+i\epsilon}$ and $S(p) = \frac{-(\not{p}+m_N)}{p^2-m_N^2+i\epsilon}$.

To evaluate the self-energy in medium, the vacuum propagators are replaced by the thermal ones. In the real-time formulation they have a 2×2 matrix structure. Here the 11-components of $\Delta(k)$ and $S(q-k)$ are used to obtain $\Pi^{11}_{\mu\nu}(q)$. This is related to the thermal self-energy function as

$$\begin{aligned} \mathrm{Im}\Pi_{\mu\nu}(q_0) &= \coth(\beta q_0/2)\mathrm{Im}\Pi^{11}_{\mu\nu} \\ \mathrm{Re}\Pi_{\mu\nu} &= \mathrm{Re}\Pi^{11}_{\mu\nu} \end{aligned} \quad (7)$$

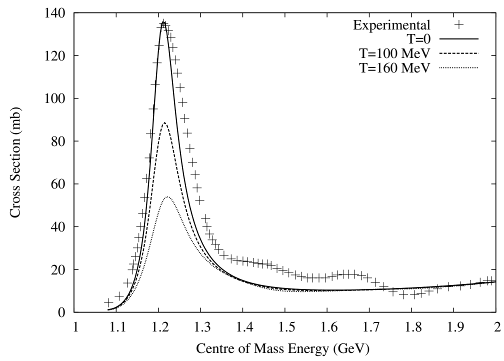

FIG. 3: The π-N scattering cross-section.

As is well-known, non-zero values of the imaginary parts of the self-energy in a given kinematic region correspond to the possibility of occurrence of specific physical processes. The contribution in the Landau region is due to absorption of the Δ due to scattering with pions and nucleons in the medium and the unitary region is due to loss by decay. These are shown in Fig. 1 upper and lower panels respectively.

The corresponding spectral functions are shown in Fig. 2. As seen from the values of the invariant mass on the x-axis, the two regions are quite nicely separated. Moreover the magnitude of the unitary contribution due to in-medium decay of the Δ is much larger.

The experimental cross section for π-N scattering shows a peak around centre of mass energy $\sqrt{s} = 1232$ MeV due to the formation of Δ resonance. Using the Lagrangian $\mathcal{L}$ as given above we evaluate the isospin averaged matrix elements introducing the self-energy in the Δ propagator. As seen in Fig. 3 this exercise reproduces the vacuum cross-section fairly well. Having thus normalised the model against experimental data we introduce the in-medium self-energy shown above. We see that the peak of the cross-section reduces substantially for higher values of temperature. For all results shown, we have used $\mu_\pi = 85$ MeV, $\mu_N = 200$ MeV and $|\vec{q}| = 500$ MeV.