Orbitally excited mass spectra of Charmonium

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Introduction

The hadron spectroscopy is interesting due to the number of states observed experimentally [1]. The theoretical explanation of orbitally excited state of charmonium is challenging because many states may have possibilities of exotic states [2–6]. Properties like mass spectra and radiative decays are influenced by relativistic effects. The mass spectra of charmonia are calculated in the framework of quark model using relativistic correction to the kinetic energy term for the potential index 0.5 to 2.0. The decay properties are also very important to know the dynamics of the hadrons[3, 7–9].

Theoretical framework

The charmonium ($c\bar{c}$)is made of c-quark and anti c-quark. We consider a Hamiltonian given by[7],

$$H = \sqrt{p^2 + m_Q^2} + \sqrt{p^2 + m_{\bar{Q}}^2} + V(r). \qquad (1)$$

where, p is the relative momentum of two quarks and V(r) is quark anti-quark potential. We have taken the quark mass parameters m_c =1.45 GeV. We use potential of the form,

$$V(r) = -\frac{\alpha_c}{r} + Ar^v + V_0. \qquad (2)$$

where α_c= $\frac{4}{3}$ α_s, α_s is the strong running coupling constant, A is the potential parameter with constant value, ν is power index,which varies from 0.5 to 2.0 and V_0 is constant. We solve,

$$H\psi = E\psi \qquad (3)$$

Using Hydrogenic wave function,

$$R(r) = \sqrt{\frac{\mu^3(n-l-1)!}{2n(n+l)!}}(\mu r)^l e^{-\mu r/2} \, L_{n-l-1}^{2l+1}(\mu r). \qquad (4)$$

Here, μ is the variational parameter and L_{n-l-1}^{2l+1} is Laguerre polynomial. For nJ state, we compute the spin average or the center of weight mass from the respective experimental values as

$$M_{CW,nJ} = \frac{\Sigma_J 2(2J+1)M_{nJ}}{\Sigma_J 2(2J+1)}. \qquad (5)$$

We add Spin dependent part separately in the Hamiltonian of Eqn.[3]. The spin orbital interaction provide the mass splitting of $J^{PC} = 0^{++}, 1^{++}\&\ 2^{++}$ states.

$$\begin{aligned} V_{SD}(r) = {} & \left(\frac{L\cdot S_Q}{2m_Q^2} + \frac{L\cdot S_{\bar{Q}}}{2m_{\bar{Q}}^2}\right) \qquad (6) \\ & \left(-\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s\frac{1}{r^3}\right) + \frac{4}{3}\alpha_s\frac{1}{m_Q m_{\bar{Q}}}\frac{L\cdot S}{r^3} \\ & + \frac{4}{3}\alpha_s\frac{2}{3m_Q m_{\bar{Q}}}S_Q\cdot S_{\bar{Q}}4\pi\delta(r) + \frac{4}{3}\alpha_s\frac{1}{m_Q m_{\bar{Q}}} \\ & \left(3(S_Q\cdot n)(S_{\bar{Q}}\cdot n) - (S_Q\cdot S_{\bar{Q}})\right)\frac{1}{r^3} \end{aligned}$$

Where V(r) is the phenomenological potential, first term the spin-orbital interaction, second term spin-spin interaction and third term is the tensor interaction. We have used virial theorem to determine the variational parameter μ for each value of the potential index.

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TABLE I: Orbitally excited P and D state masses of $c\bar{c}$ mesons (in GeV).

ν	1^1P_1	1^3P_0	1^3P_1	1^3P_2	1^1D_2	1^3D_1	1^3D_2	1^3D_3	2^1P_1	2^3P_0	2^3P_1	2^3P_2	2^1D_2	2^3D_1	2^3D_2	2^3D_3
0.5	3.27	3.21	3.25	3.29	3.41	3.40	3.41	3.42	3.45	3.35	3.41	3.50	3.56	3.53	3.56	3.58
0.7	3.32	3.24	3.27	3.36	3.52	3.50	3.51	3.53	3.59	3.42	3.53	3.66	3.75	3.70	3.73	3.78
0.9	3.39	3.28	3.35	3.42	3.63	3.61	3.62	3.64	3.74	3.51	3.65	3.83	3.95	3.89	3.94	3.99
1.0	3.41	3.30	3.37	3.45	3.69	3.68	3.68	3.70	3.82	3.55	3.71	3.93	4.07	4.00	4.05	4.11
1.1	3.44	3.32	3.40	3.49	3.75	3.73	3.75	3.76	3.90	3.60	3.78	4.03	4.18	4.11	4.16	4.23
1.3	3.50	3.32	3.47	3.62	3.87	3.87	3.87	3.88	3.07	3.68	3.91	4.24	4.43	4.34	4.40	4.48
1.5	3.56	3.41	3.50	3.63	4.00	4.01	4.00	4.00	4.24	3.76	4.04	4.45	4.69	4.58	4.65	4.75
1.7	3.60	3.39	3.50	3.70	4.13	4.17	4.13	4.11	4.42	3.84	3.18	4.68	4.96	4.85	4.92	5.03
1.9	3.67	3.52	3.60	3.77	4.26	4.34	4.26	4.23	4.61	3.93	4.31	4.92	5.24	5.13	5.19	5.39
2.0	3.70	3.53	3.60	3.79	4.31	4.41	4.31	4.26	4.69	3.96	4.36	5.03	5.37	5.27	5.31	5.45
Exp.	3.52	3.41	3.51	3.56		3.78	3.82			3.92			4.19	4.15		
[4]	3.52	3.41	3.51	3.55	3.80	3.78	3.79	3.81	3.93	3.87	3.90	3.95	4.20	4.15	4.19	4.22
[5]	3.51	3.45	3.50	3.53	3.81	3.79	3.81		3.95	3.91	3.94	3.97	4.16	4.15	4.16	
[6]	3.52	3.44	3.51	3.55	3.84	3.82	3.84		3.96	3.92	3.95	3.98	4.21	4.19	4.21	

Result and discussion

The P and D wave masses of $c\bar{c}$ are calculated using the quark anti-quark potential of the form Coulomb plus power potential. The relativistic correction to the kinetic energy term are also incorporated. The Spin dependent part is added separately Eqn.-(3) for the mass of the P and D state of the charmonium. The calculated masses for potential index 0.5 to 2.0 are listed in Table-1. Details of the results will be presented in the conference.

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