Mass spectrum of pseudoscalar and vector $c\bar{c}$ and $b\bar{b}$ systems

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Introduction: In this work we have calculated the mass spectrum of ground and radially excited states of pseudoscalar charmonium and bottomonium such as η_c and η_b, as well as J/ψ, and ¥, respectively. Such studies have become a hot topic in recent years, due to observation of many new states at various high energy accelerators at BABAR, Belle, CLEO and BES-III collaborations [1]. All this has opened up new challenges in theoretical understanding of heavy hadrons and provide an important tool for exploring the structure of these simplest bound states in QCD and for studying the non-perturbative (long distance) behavior of strong interactions. We employ the formulation of Bethe-Salpeter equation under Covariant Instantaneous Ansatz (CIA), which is a Lorentz-invariant generalization of Instantaneous Approximation. We employ a 4X4 representation for two-body quark-anti-quark BS amplitude for calculating both the mass spectra as well as the transition amplitudes. However, the price we have to pay is to solve a coupled set of equations for both pseudoscalar and vector quarkonia. However, in the heavy quark approximation, we have shown that these equations can indeed be decoupled, and lead to algebraic expressions for the mass spectral equations [2], leading to analytical solutions for both masses, as well as eigen functions, in an approximate harmonic oscillator basis, and thus leading to a deeper understanding of this problem (for details see Ref.[2]).

This is unlike the other works using BSE, that from the beginning, employ numerical solution of the coupled set of equations to calculate the spectra. Further, in the present framework of BSE, the hadron-quark vertex function (and the full 4D BS wave function) for pseudoscalar and vector quarkonia used for calculation of various processes, accommodates all the Dirac structures from their complete set in a natural manner, employing the power counting rule [2-4] we had proposed recently.

Mass Spectrum: We first start with a BSE with a 4X4 representation of quark-anti-quark BS amplitude for a hadron of momentum P, and mass M, with constituent quarks of masses m_1 and m_2, interacting through an interaction kernel K, which is assumed to depend on the component $\hat{q}$ of internal momentum (q of the hadron), that is transverse to the external hadron momentum P. For pseudoscalar (P), and vector (V) mesons, the instantaneous BS wave functions under CIA incorporating the full Dirac structure are expressed as [2-4]:

$$\psi^P(\hat{q}) = N_P[M + \gamma.P + \frac{\gamma.\hat{q}}{M}\gamma.P]\gamma_5\varphi^P(\hat{q});$$

$$\psi^V(\hat{q}) = N_V[M\gamma.\varepsilon + \frac{M}{m}\hat{q}.\varepsilon + \gamma.\varepsilon\gamma.P + \frac{1}{m}\gamma.P\hat{q}.\varepsilon - \frac{1}{m}\gamma.P\gamma.\varepsilon\gamma.\hat{q}]\varphi^V(\hat{q}),$$

respectively. The mass spectrum of ground (1S) and radially excited (2S, 3S,…) states for pseudoscalar (0^{-+}) and vector (1^{--}) quarkonia in an approximate harmonic oscillator basis is derived as [2]:

$$\frac{1}{2\beta_P^2}\left[\frac{M^2}{4} - m^2 + \frac{C_0\beta_P^4}{\omega_0^2}\left[1 + 2A_0\left(N + \frac{3}{2}\right)\right]^{\frac{1}{2}}\right] = N + \frac{3}{2}$$

$$\frac{1}{2\beta_P^2}\left[\frac{M^2}{4} - m^2 + \frac{C_0\beta_P^4}{\omega_0^2}\left[1 + 2A_0\left(N + \frac{3}{2}\right)\right]^{\frac{1}{2}}\right] = N + \frac{3}{2}$$

respectively, where N=0,2,4,…., where the inverse range parameters are [2]:

$$\beta_P = [4\frac{m\omega_{q\bar{q}}^2}{\sqrt{1+2A_0\left(N+\frac{3}{2}\right)}}]^{\frac{1}{4}};\ \beta_V = [3\frac{m\omega_{q\bar{q}}^2}{\sqrt{1+2A_0\left(N+\frac{3}{2}\right)}}]^{\frac{1}{4}}$$

where the factors 4 (in P-mesons) and 3 (in V-mesons), come from the spin-spin interactions. The mass spectrum calculated in this BSE framework for

(1S,...,4S) states of η_c and η_b, as well as (1S,...,6S) states of J/ψ, and ¥, are shown in Tables 1 and 2 [2] respectively. All numerical calculations have been done using Mathematica. We first fit our parameters to the ground state masses of all these states. These input parameters were found to be C_0=0.29, ω_0=0.158 GeV., QCD length scale, Λ=0.200GeV, and A_0=0.01. With input quark masses m_c=1.483 GeV., and m_b= 4.990 GeV, we obtained the best fit to these ground state masses [2]. The same set of parameters above was used to calculate the masses, and analytic forms of wave functions of all the other (radially excited) states of η_c, η_b, as well as J/ψ, and ¥ mesons.

	BSE-CIA	Expt.[1]
η_c(1S)	2.9814	2.983±.0007
η_c(2S)	3.5733	3.639±.0013
η_c(3S)	4.0716	
η_c(4S)	4.5081	
η_b(1S)	9.1645	9.398±.0032
η_b(2S)	9.7603	9.999±.0028
η_b(3S)	10.3118	
η_b(4S)	10.8265	

Table 1: Masses of ground and radially excited states of η_c and η_b (in GeV.) in (BSE-CIA) along with experimental data.

The masses and eigen functions thus calculated are used for calculating other processes [2] for ground and radially excited states of these pseudoscalar and vector quarkonia.

	BSE-CIA	Expt.[1]
J/ψ(1S)	3.0751	3.0969±0.000011
J/ψ(2S)	3.4927	3.6861±0.00034
J/ψ(3S)	3.8591	4.030± 0.001
J/ψ(4S)	4.1881	4.421±0.004
J/ψ(5S)	4.4885	
¥(1S)	9.6735	9.4603±0.00026
¥(2S)	10.0771	10.0233±0.00031
¥(3S)	10.4582	10.3552±0.00005
¥(4S)	10.8198	10.5794±0.0012
¥(5S)	11.1640	10.865±0.008
¥(6S)	11.4927	11.019±0.008

Table 2: Masses of ground and radially excited states of J/ψ and ¥ (in GeV.) in (BSE-CIA) along with experimental data.

The results obtained for masses of ground and radially excited states of η_c, η_b, J/ψ, and ¥ mesons are in good agreement with experiment. Some of our results are within the error bars of data [1]. In this first stage of our framework that makes use of 4X4 BSE from the beginning, our main emphasis was to obtain algebraic form of mass spectral equation, and obtain its analytical solutions for both masses, as well as the wave functions (for l=0 states), and use them to calculation of leptonic decay constants, two-photon decays, and two-gluon decays in an approximate harmonic oscillator basis [2]. This framework will now be extended to incorporate l=2,4,.. states as well. We also intend to extend this study to calculation of observables of heavy-light mesons such as D and B, and also to processes involving quark-triangle diagrams with two or more hadronic vertices such as to decays $V \rightarrow P\gamma$, and $V \rightarrow PP$ (with P and V being the pseudoscalar and vector quarkonia respectively), as further works.

References:

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