Multifractal analysis of relativistic charged particle distribution in ³²S-AgBr interactions at 200 AGeV.

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Introduction

The study of non-statistical fluctuations in relativistic nuclear collisions has recently attracted a great deal of attention due to the possibility of extracting important information about the mechanism of multiparticle production in such collisions. Takagi [1] proposed a new method for studying the multifractal structure of multiparticle production and successfully applied this methodology to probe fractality in UA5 data on proton-antiproton interactions [2] and TASSO and DELPHI data on electron positron annihilations [3, 4]. It has been pointed out by Takagi that the deviations from the linear behaviour in a log-log plot may be partly due to the fact that the above methods are unable to give the required mathematical limit: the number of points tending to infinity.

In this paper an attempt has been made to study the multifractal nature of multiplicity distributions of produced pions in pseudo-rapidity space in ³²S-AgBr interactions at 200 AGeV by the method enunciated by Takagi. It would be interesting to study the fractal nature at such energy. We have calculated the multifractal specific heat of the produced pions from the variation of D_q with lnq/(q-1) for our data. The experimental results have been compared with those obtained by analyzing events generated with the computer code FRITIOF based on Lund Monte Carlo model [5] for high energy nucleus-nucleus interaction.

Experimental Technique

In this experiment two stacks of Ilford G5 nuclear emulsion plates exposed horizontally to a ³²S-beam at 200 AGeV obtained from Supper Proton Synchrotron (SPS) at CERN have been utilized for data collection. The other relevant details about the present experiment can be found in our earlier publication [6].

Results and Discussions

In order to investigate the multifractality of relativistic shower particles produced in ³²S-AgBr interactions at 200 AGeV in η – space, the analysis of the multiplicity distribution in the central pseudo-rapidity region $(\eta_{peak} - 1.5 < \eta < \eta_{peak} + 1.5)$ has been considered, which covers most of the produced shower particles. The initial $\Delta \eta = 3.0$ was subsequently reduced in steps of 1. The values of $< n \ln n > / < n >$ and $< n^q >$ for present data are computed for each interval and are shown in Fig. 1. The values of $ln < n^q >$ as a function of ln < n > for different q are also shown in Fig. 1 for ³²S-AgBr collisions at 200 AGeV. All the experimental points clearly follow an excellent linear relation in each plot for the whole range of <n>. The errors shown in the plots are purely statistical. The linear behaviour of $ln < n^q$ with ln < n in the figure gives an indication of fractal structure in multiparticle production in η -space. Similar observations have also been reported by other workers [7-9].

For the sake of comparison with the experimental data, the results due to the FRITIOF generated events are also plotted in Fig. 1 for ³²S-AgBr interactions. The observed points follow an excellent linear relation for the whole range of $\langle n \rangle$ similar to the experimental data. Hence, it has been found that the experimental data on multifractality exhibit a remarkable closeness to analogous data obtained from the FRITIOF model. The results therefore contemplates that there is a correspondence between the experimental data and the results for events generated using the FRITIOF model.

The values of the generalized dimensions are computed from the best fitted slopes of linear relation of $ln < n^q >$ with ln < n > for both experimental and FRITIOF data and are plotted in Fig.2. It is easily noticed from the figure that the values of generalized dimensions, D_q



Fig. 1 Plot of $ln < n^q$ and nln < n > /<n > against ln < n > for ³²S-AgBr Interactions at 200 AGeV along with the FRITIOF Data.

decreases with increasing order of moment, q, thereby showing multifractality in pion production for all the interactions, which in turn supports an interpretation in terms of a cascade mechanism in the multiparticle production process in pseudo-rapidity phase space. So the present analysis of the multifractal structure shows a remarkable property of the observed fluctuations.



Fig. 2 Variation of generalized dimensions with the order of moments, q.

Bernoulli type of fluctuations are responsible for a transition from monofractality to multifractality, Bershadskii [10] gave a thermodynamic interpretation of the observed results in terms of a constant specific heat c, $D_q = D_{\infty} + c \ln q / (q-1)$. The slope c in the above equation can be interpreted as the multifractal specific heat of the system provided that the thermodynamical interpretation of multifractality is used. By plotting D_a against lnq/(q-1) we can therefore, obtain the value of specific heat from the slope of best linear fit. Such a plot is shown in Fig. 3 for the experimental and FRITIOF simulated values. The calculated multifractal specific heat extracted from the figure is given in Table 1 for experimental and the corresponding the FRITIOF simulated value. Moreover, the results obtained by other workers [7-9] are included in the table for comparison. So, it is observed from the table that in pseudorapidity phase space the constant specific heat approximation as given by Bershadskii is reasonable for nucleus-nucleus interactions.



Fig. 3 Plot of D_q against lnq/(q-1) **Table 3: Values of multifractal specific heats.**

Interactions	Energy	Specific heat,	References
	(AGeV)	с	
³² S-AgBr	200	0.28 ± 0.019	Present
		(0.31±0.01)*	work
24N (4 5	0.21 + 0.01	[7]
Mg-AgBr	4.5	0.31 ± 0.01	[/]
¹² C-AgBr	4.5	0.29 ± 0.03	[8]
²⁸ Si-AgBr	14.5	0.30 ± 0.03	[8]
¹⁶ O-AgBr	60	0.21 ± 0.01	[9]
*FRITIOF Data			

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