# Radial Flow in Non-Extensive Thermodynamics and Study of Particle Spectra at LHC in the Limit of Small $(q-1)$

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## Introduction

Tsallis distribution [1] gives excellent fits to the transverse momentum distributions observed at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC) with only three parameters q, Tsallis temperature T and volume V. The parameter q is referred to as the Tsallis parameter. The form of the Tsallis distribution has been described in details previously [2] and has the advantage of being thermodynamically consistent. It is quite remarkable that the values obtained for the parameters $q$ and $T$ are consistent with each other for different particle species. This supports the picture that $p+p$ collisions at high energies produce a final-state system which is consistent with obeying Tsallis non-extensive thermodynamics.

## Taylor expansion of Tsallis statistics and transverse momentum spectra

The Tsallis Boltzman distribution function is given by,

$$f = \left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}} \tag{1}$$

In case of high energy physics, q value is close to 1, so for simplicity we expand $f^q$ about q=1, which is given by,

$$\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} \simeq \mathrm{e}^{-\frac{E-\mu}{T}}\left\{1+(q-1)\frac{1}{2}\frac{E-\mu}{T}\left(-2+\frac{E-\mu}{T}\right)+\mathcal{O}(q-1)^2..\right\}. \tag{2}$$

Here the first term is the Boltzmann distribution and rest are higher orders in (q-1).

The invariant yield using the Taylor expansion is given by,

$$\frac{dN}{p_T dp_T dy d\phi} \simeq \mathcal{C}Ee^{-\Phi} + \mathcal{C}E\frac{x}{1!}\frac{\Phi}{2}\left(-2+\Phi\right) e^{-\Phi} + \mathcal{C}E\frac{x^2}{2!}\frac{\Phi^2}{12}\left(24-20\Phi+3\Phi^2\right)e^{-\Phi} \tag{3}$$

where $\phi = \frac{E-\mu}{T}$, $x=(q-1)$ and C $= \frac{gV}{2\pi^3}$

## Inclusion of radial flow

In order to see how the inclusion of radial flow could improve the description of the transverse momentum spectra obtained in heavy-ion collisions, we have taken a cylindrical symmetry in which it has a explicit dependence on flow parameter $\beta$. Here we have replaced $f(E)$ by $f(p^\mu u_\mu)$, where $(p^\mu u_\mu)$ is a Lorentz invariant quantity and

$$p^\mu = (m_T cosh y,\ p_T cos\phi,\ p_T sin\phi,\ m_T sinh y)$$
$$u_\mu = (\gamma cosh\xi,\ \gamma v cos\alpha,\ \gamma v sin\alpha,\ \gamma sinh\xi)$$

where $\xi$ is the space-time rapidity.

However, the invariant yield including flow up to first order in (q-1) is given by,

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$$\frac{1}{p_T}\frac{dN}{dp_T dy} = \frac{gV}{(2\pi)^2} \Big\{ 2T[rI_0(s)K_1(r) - sI_1(s)K_0(r)] -(q-1)Tr^2I_0(s)[K_0(r)+K_2(r)] +4(q-1)\ TrsI_1(s)K_1(r) -(q-1)Ts^2K_0(r)[I_0(s)+I_2(s)] +\frac{(q-1)}{4}Tr^3I_0(s)[K_3(r)+3K_1(r)] -\frac{3(q-1)}{2}Tr^2s[K_2(r)+K_0(r)]I_1(s) +\frac{3(q-1)}{2}Ts^2r[I_0(s)+I_2(s)]K_1(r) -\frac{(q-1)}{4}Ts^3[I_3(s)+3I_1(s)]K_0(r) \Big\} \quad (4)$$

where $r \equiv \frac{\gamma m_T}{T}$, $s \equiv \frac{\gamma v p_T}{T}$
$I_n(s)$ and $K_n(r)$ are the modified Bessel functions of the first and second kind respectively.

## Results and discussion

In Fig. 1 we show fits to the normalized differential $\pi^-$ yields in $(0-5)\%$ Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV, with the Tsallis (solid line) and Boltzmann distributions (dashed line). Also shown are fits with the Tsallis distribution keeping terms to first order (dash-dotted line) and second order in $(q-1)$ (dotted line). The lower part of the figure shows the difference between the Tsallis distribution (M) and experiment (E). It is clear that the best fit is achieved with the full Tsallis distribution, whereas, using the Boltzmann distribution the description is not good. Successive corrections in $(q-1)$ improve the description. There is a clear deviation at very low transverse momentum (below 0.5 GeV) and also at higher values above 2.75 GeV.

In Fig.2 we have fitted the transverse momentum spectra of Pb+Pb at $\sqrt{s}_{NN} = 2.76$ TeV with Tsallis distribution including radial flow up to first order in $(q-1)$ (dashed line) up to 3 GeV. The parameters obtained from fitting the transverse momentum spectra are $\beta = 0.609$, with $T = 0.146$ MeV, $q = 1.030$ and the radius of the volume is $R = 29.8$ fm.

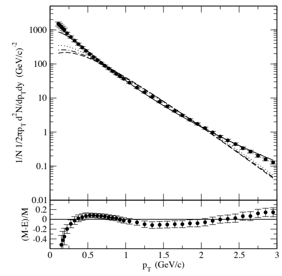

FIG. 1: Fits to the normalized differential $\pi^-$ yields in $(0-5)\%$ Pb-Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV.

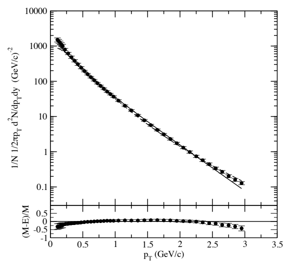

FIG. 2: Fits to the normalized differential $\pi^-$ yields in $(0-5)\%$ Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV.

Using Taylor expansion method, we have studied the degree of deviation of Tsallis thermodynamic variable from thermalized Boltzmann distribution[3]. Using the Taylor expansion, we included flow in the Tsallis distribution, but this is a coarse way of treating flow. More elaborate descriptions can be obtained by including the Cooper-Frye freeze-out parameters.

## References

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