Multifractal Detrended fluctuation analysis of Ring like events at CERN SPS Energy

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Introduction

Studying or probing highly excited nuclear dense matter under controlled conditions in the laboratory has proven their worth in exploring the nature of matter in extreme conditions of temperature and density. Under such extreme conditions a new form of matter called Quark Gluon Plasma (QGP) is created **[1]**. In the analyses of azimuthal distributions of produced pions, two different classes of substructures were revealed, which could be referred to as jet-like and ring-like structures. Ring-like structures are observed in the distribution of particles if those are clustered in the narrow region of pseudorapidity (η),but distributed more or less uniformly over the whole azimuth(ϕ) like the spokes of a wheel. Ring like structures are first observed in cosmic ray experiment. Most of the attempts have been made to study the fractal nature in multiparticle production of pions in ring-like and jet-like events [2-3].

In recent years much effort has been devoted to reliable identification of the multifractality in real data coming from such various fields like, e.g., DNA sequences , physiology of human heart, neuron spiking , atmospheric science and climatology, financial markets, geophysics and many more. Multifractal detrended fluctuation analysis MFDFA [4], is based on the identification of scaling of the *q*th-order moments that power-law depend on the signal length and is a generalization of the standard DFA using only the second moment *q*=2. In recent years the detrended fluctuation analysis (DFA) method **[5]** has become a widely used technique for the determination of (mono-) fractal scaling properties and the detection of long-range correlations in noisy, non-stationary time series **[6]**. In this case a multitude of scaling exponents is required for a full description of the scaling behavior, and a multifractal analysis must be applied. In general, two different types of multifractality in time series can be distinguished: (i) Multifractality due to a broad probability density function for the values of the time series. In this case the multifractality cannot be removed by shuffling the series. Multifractality due to different long-range (time-) correlations of the small and large fluctuations. In this case the probability density function of the values can be a regular distribution with finite moments, e.g. a Gaussian distribution.

In this paper we employ the DFA and the MF-DFA methods to analyze the pseudorapidity (η) distribution of ring like events of charged mesons produced in ^{32}S + Ag/Br interaction at 200A GeV to get an idea of the underlying phase space structure in that interaction.

Methodology

The multifractal generalization of this procedure MFDFA can be briefly sketched as follows. First, for a given time series {x(i); i=1……N} on a compact support, one calculates the integrated signal profile $Y(j) = \sum_{i=1}^{j}(x(i) - \langle x \rangle), j = 1 \dots N$. Where <…> denotes averaging over the time series, and then one divides it into M_n segments of length n (n <N) starting from both the beginning and the end of the time series (i.e., $2M_n$ such segments total). Each segment ν has its own local trend that can be approximated by fitting an 1st order polynomial $P_\nu^{(l)}$ and subtracted from the data; next, the variances for all the segments ν and all segment lengths n must be evaluated $\boldsymbol{F^2(\nu, n) = \frac{1}{n}\sum_{j=1}^{n}\{Y[\nu - 1)n + j] - P_\nu^{(l)}(j)\}^2}$.Finally, $\boldsymbol{F^2(\nu, n)}$ is averaged over ν's and the q^{th}-order fluctuation function is calculated for all possible segment lengths n:

$$\boldsymbol{F_q(n) = (\frac{1}{2M_n}\sum_{\nu=1}^{2M_n}[F^2(\nu, n)]^{q/2})^{1/q}, \; q \epsilon R} \text{--(1)}$$

The key property of $F_q(n)$ is that for a signal with fractal properties, it reveals power-law scaling within a significant range of n, $\boldsymbol{F_q(n) \sim n^{h(q)}}$**-(2)**
The result of the MFDFA procedure is the family of exponents h(q) (called the generalized Hurst exponents). For very large scales, n>N=4; $F_q(n)$ becomes statistically unreliable because the number of segments M_n for the averaging procedure becomes very small. Thus, we usually exclude scales n>N/4 from the fitting procedure to determine h(q). The value of h(0), which corresponds to the limit h(q) for q → 0, cannot be determined directly using the averaging procedure in Eq. (1) because of the diverging exponent. Instead, a logarithmic averaging procedure has to be employed,

$$\boldsymbol{F_0(n) \equiv \exp\{\frac{1}{4M_n}\sum_{\nu=1}^{2M_n} ln\,[F^2(\nu,n)]\} \sim n^{h(0)}}$$

For an actual multifractal signal, form a decreasing function of q, while for a monofractal h(q)=const **[4]**.

Results & Discussions

We have shown the variation of Logarithmic variation of generalized fluctuation functions $\boldsymbol{F_q(n)}$ with log of scale **n** for q=±5,±4.5, ±4,±3.5,±3,±2.5,±2,±1.5,±1,±0.5,±0.4,±0.3,±0.2, ±0.1 & 0 in **Fig.1**. On large scales n, we observe the expected power-law scaling behavior according to Eq. (2), which corresponds to straight lines in the log–log plot.

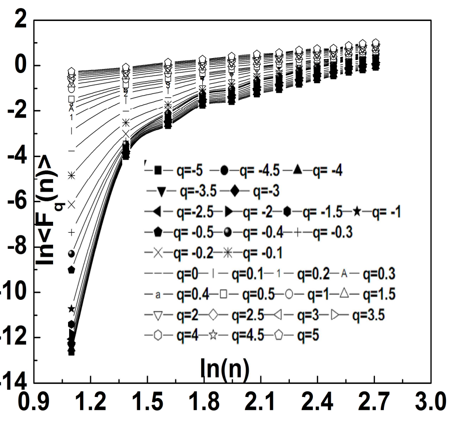
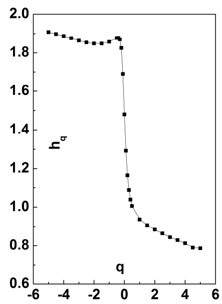


Fig.1

In **Fig.2**, the scaling exponents h(q) determined from the slopes of these straight lines are shown versus q. Clearly, q dependency is observed,



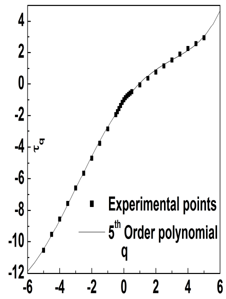


Fig.2 Fig.3

Scaling exponent, **[1]** τ(q) =qh(q)-1 has also been plotted in **Fig.3**. We fitted the experimental points with 4th order polynomial and it is clear that τ(q) is showing a strong nonlinear dependency with q.
From the above analyses it has been found that h(q) is showing a q dependency which may be a signal of multifractality of the phase space structure of particle production. At the same time we also observed a nonlinear q dependency of τ(q) indicating multifractality of phase space structure. However, a definite conclusion cannot be drawn from the above analyses about the phase space structure, this topic needs serious investigation.

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