Study of Levy stable law in compound multiplicity distribution under self-affine scaling scenario

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One of the primary motivations of high energy experiments is to search for the signal of QGP phase transition. Up to now many possible signals about the phase transition have been proposed theoretically. However, the formation of QGP as well as the signals of such phase transition is under debate. Thus further studies about the signal are needed. In order to detect the existence of phase transition in hadronization process the Levy stable law [1] has been used. This law is characterized by the Levy stability index μ which has a continuous spectrum within the region of stability [0, 2]. The index, μ, allows an estimation of cascading rate. μ =2 corresponds to the minimum fluctuations from self-similar branching processes and μ =0 corresponds to the maximum fluctuations which characterizes the interacting system as monofractal. But phase transition cannot be indicated by monofractal behaviour alone. A thermal phase transition occurs when $\mu \langle 1$ and when $\mu \rangle 1$, there is a non-thermal phase transition during the cascading process.

It is generally believed that medium energy knocked out protons, which manifest themselves as grey particles in nuclear emulsion, are supposed to carry relevant information about the hadronization mechanism, since the time scale of emission of these particles is of the same order as that of the produced shower particles (pions). One may consider pions and medium energy protons together in equal footings without making any distinction between them and termed them as “compound hadrons”. Therefore if one combines the number of grey (n_g) and shower particles (n_s) per event in a collision as a new parameter, named “compound multiplicity” ($n_c = n_g + n_s$), it could also play an important role in understanding the reaction dynamics in high energy nuclear interactions. Here the phase transition phenomenon has been studied thoroughly using the Levy stable law for the produced compound hadrons and pions separately for $\pi^- - AgBr$ interactions at 350 GeV in two dimensional $(\cos\theta - \phi)$ phase space under self-affine scenario.

Factorial moment of order q is defined as [2] $F_q(\delta x_1, \delta x_2)$

$$= \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)........(n_m-q+1)\rangle}{\langle n_m\rangle^q},$$

n_m is the multiplicity in the m th cell. M is the number of two-dimensional cells into which the considered phase space has been divided. Here M_1 is the number of bins along x_1 direction and M_2 is the number of bins along x_2 direction. The shrinking ratios along x_1 and x_2 directions are characterised by a parameter $H = \ln M_1 / \ln M_2$ where $0 < H \le 1$ is called the Hurst exponent. H <1 signifies that the phase spaces are divided anisotropically, consequently the fluctuations are self affine in nature. From the power law dependence of factorial moment on the cell size as cell size approaches to zero, the index α_q is obtained from a linear fit of the form $\ln\langle F_q\rangle = \alpha_q \ln M + a$, where a is a constant. Now a quantity β_q is defined as $\beta_q = \frac{d_q}{d_2}(q-1)$ and β_q is related to Levy index (μ) by the equation $\beta_q = \frac{\alpha_q}{\alpha_2} = \frac{q^{\mu} - q}{2^{\mu} - 2}$.

In order to reduce the effect of non-flat average distribution, the cumulative variables $X_{\cos\theta}$ and X_{ϕ} are used instead of $\cos\theta$ and ϕ. In the $X_{\cos\theta}$- X_{ϕ} space we divided the region [0, 1] into $M_{\cos\theta}$ & M_{ϕ} bins respectively. The partitioning was taken as $M_{\cos\theta} = M_{\phi}{}^{H}$. We choose the partition number along ϕ direction as M_{ϕ} = 10,…..,50. The ($X_{\cos\theta}$-X_{ϕ}) space is divided into M = $M_{\cos\theta} \times M_{\phi}$ cells and calculation is done in each bin independently. The full bin range is divided into different sub-bin ranges e.g. $10 \le M_{\phi} \le 20$, $20 \le M_{\phi} \le 30$, $30 \le M_{\phi} \le 40$, $40 \le M_{\phi} \le 50$. In the ($X_{\cos\theta}$-X_{ϕ}) phase space the anisotropic behavior of compound hadrons is best revealed at different H values for different bin ranges which are tabulated in Table 1. For these H values the plot of $\ln\langle F_q \rangle$ as a function of $\ln M$ gives the value of α_q. Using the values of α_q, values of β_q are calculated. It is observed that the parameter β_q increases with increasing order of moments. This indicates the fact that compound multiplicity distribution has multifractal structure. Therefore, we can say that hadrons in the final state are produced as a result of a self-similar cascade mechanism. From values of β_q the Levy index obtained for the $X_{\cos\theta}$- X_{ϕ} space are listed in Table 1 which are within the permissible limit $0 \le \mu \le 2$. Here $\mu \rangle$ 1 would have indicated a non- thermal phase transition during the cascade process. The whole procedure is repeated for shower multiplicity distribution also. Corresponding values are calculated and tabulated in Table 1. An evidence of thermal phase transition is obtained for self-affine fluctuations of pions.

Table 1: Values of different parameters

Multiplicity	Bin range	H	μ
Compound	$10 \le M_{\phi} \le 20$	0.7	1.120 ± 0.006
	$20 \le M_{\phi} \le 30$	0.7	1.524 ± 0.005
	$30 \le M_{\phi} \le 40$	0.7	1.297 ± 0.002
	$40 \le M_{\phi} \le 50$	0.3	1.110 ± 0.002
Shower	$10 \le M_{\phi} \le 20$	0.7	0.535 ± 0.005
	$20 \le M_{\phi} \le 30$	0.4	0.914 ± 0.002
	$30 \le M_{\phi} \le 40$	0.6	0.625 ± 0.002
	$40 \le M_{\phi} \le 50$	0.4	0.493 ± 0.004

References

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