Bulk viscosity of hot and dense hadrons

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Introduction

The energy momentum tensor T_{ij} in the local rest frame (flow velocity $u^\mu = \{1,0,0,0\}$) for ideal fluid is given by

$$T_{ij}\,(\epsilon, n) \;=\; P(\epsilon, n)\delta_{ij} \tag{1}$$

where $P\,(\epsilon, n)$ is the thermodynamic pressure of the system and is a function of the energy density ϵ and number density n. This relationship between P, ϵ and n is known as the equation of state (EoS). In a dissipative system, transport coefficients are introduced by parametrising the system's response to the gradients of various thermodynamic quantities. Two such transport coefficients, namely shear (η) and bulk (ζ) viscosities appear as the coefficients of first order in derivatives of fluid velocity. In the present contribution we focus mainly on the bulk viscosity of hot and dens hadronic gas.

Bulk Viscosity

A fluid in equilibrium can fluctuate to a non-equilibrium state in many ways. Depending on the mode of fluctuation there is an onset of the corresponding dissipative process [1]. Within the ambit of linear response theory, the medium response allows us to compute the transport coefficients like η, ζ etc. e.g., in presence of non-zero velocity gradients of the kind $\frac{\partial u_i}{\partial x_j}$ with $i \neq j$, η comes into play. In order to compute ζ, we need to turn on $\nabla \cdot u$. In this work we will consider the contribution to ζ due to change in particle number alone. We compute ζ by considering particle number fluctuation within a grand canonical ensemble (GCE) and estimate the change in the T_{ij}, δT_{ij} due to fluctuation in particle numbers. Thus this approach is expected to be valid when macroscopically the chemical composition of the fireball has frozen and one can associate a hadron chemical potential μ_i with the ith hadron. Reading off the coefficient from δT_{ij} proportional to $\nabla \cdot u$ will allow us to estimate ζ upto a relaxation time constant.

For slowly varying equilibrium particle number N_{eq} with relaxation time scale τ_{R}, the change in N_{eq}, δN can be written as

$$\delta N = -\tau_{\mathrm{R}} \frac{\partial N_{eq}}{\partial t} \tag{2}$$

This is expected to give rise to the following change in P, δP

$$\delta P = \left(\frac{\partial P}{\partial n}\right)_\epsilon \frac{\delta N}{\Omega} \tag{3}$$

Assuming adiabaticity we have

$$\begin{aligned} \frac{dS}{dt} &= 0 \\ \Rightarrow \frac{\partial s}{\partial t} &= -s\nabla \cdot u \end{aligned}$$

Hence, the rate of change of particle number becomes

$$\frac{\partial N_{eq}}{\partial t} \;=\; -\Omega s \frac{\partial n}{\partial s} \nabla \cdot u$$

Putting above equations, we get

$$\begin{aligned} \delta P &= \left(\frac{\partial P}{\partial n}\right)_\epsilon \frac{\partial n}{\partial s} s\tau_{\mathrm{R}} \nabla \cdot u \\ \frac{\zeta}{s\tau_{\mathrm{R}}} &= \left(\frac{\partial P}{\partial n}\right)_\epsilon \frac{\partial n}{\partial s} \end{aligned} \tag{4}$$

τ_R is estimated from

$$\tau_R \;=\; \frac{1}{n\sigma\langle v\rangle} \simeq \frac{\langle\langle E\rangle\rangle}{n\sigma\langle\langle p\rangle\rangle} = \frac{1}{\sigma}\frac{1}{\sum_i n_i \frac{\langle p_i\rangle}{\langle E_i\rangle}} \tag{5}$$

where $\langle v\rangle = \langle \frac{p}{E}\rangle \simeq \frac{\langle p\rangle}{\langle E\rangle}$.

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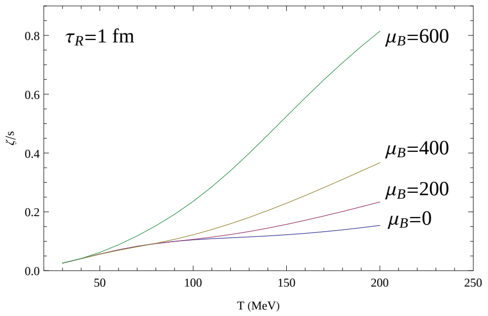

FIG. 1: Variation of the bulk viscosity to entropy density ratio with temperature for various values of baryonic chemical potential, μ_B.

Results

Eq. 4 may be used used to estimate the bulk viscosity to entropy density ratio if τ_R is assumed. We take τ_R = 1 fm/c here. The right had side of the Eq. 4 is estimated within the ambit of hadronic resonance gas model [2]. In Fig. 1 we display the variation of bulk viscosity to entropy density ratio as a function of temperature.The ratio increases with temperature as well as with baryonic chemical potential. The increase is steeper for higher baryonic chemical potential. Therefore, the bulk viscosity will play dominant role at low energy collisions (like FAIR, NICA) than collisions at higher energies (like RHIC and LHC where μ_B is small). The results for high baryonic chemical potential will be important for studying the fireball properties in the upcoming experiments at FAIR and NICA.

In Fig. 2 the ratio of bulk to shear viscosity is depicted as a function of temperature.The ratio remains above the AdS/CFT bound.

Summary

The bulk viscosity of hot and dense hadrons has been estimated within the framework of hadronic resonance gas model. We observe that the bulk viscosity to entropy ratio increases faster with temperature for higher μ_B. The magnitude of ζ is more at high μ_B. This results will have crucial importance for fireball produced at low energy nuclear collisions (FAIR, NICA) We note that the bulk to shear viscosity ratio remains above the bound set by AdS/CFT.

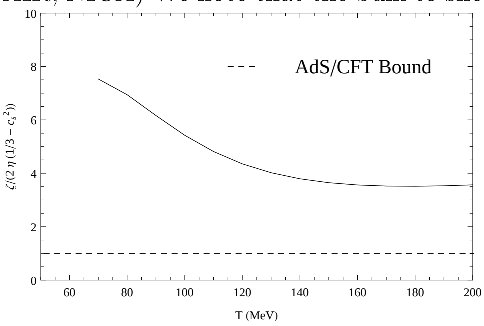

FIG. 2: Variation of the ratio of bulk to shear viscous coefficients with temperature.

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