QGP-Hadron Phase Structure in a statistical model using Cornell, Richardson and Peshier potentials

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I. INTRODUCTION

Ultra Relativistic Heavy Ion Collisions (URHIC) have always been a scope for the formation of Quark-Gluon Plasma (QGP) droplet (fireball) [1]. Albeit core QCD is an appropriate theory to understand the new deconfined matter, but phenomenological models can also pave the way to decipher the matter [2] in coordination with the ongoing experiments.

The nucleation process is driven by statistical fluctuations which produce the QGP droplets in a hadronic phase,the size of the fluctuations being determined by the critical free energy difference between the two phases.The Kapusta et al model [3,8] uses the liquid drop model expansion for this,as given by

$$\Delta F = \frac{4\pi}{3}R^3[P_{had}(T,\mu_B) - P_{q,g}(T,\mu_B)] + 4\pi R^2\sigma + \tau_{crit}T\ ln\left[1 + (\frac{4\pi}{3})R^3 s_{q,g}\right]. \quad (1)$$

The first term represents the volume contribution,the second term is the surface contribution where σ is the surface tension,and the last term is the so called shape contribution.The shape contribution is an entropy term on account of fluctuations in droplet shape which we may ignore in the lowest order approximation.The critical radius R_c can be obtained by minimising (1) withrespect to the droplet radius R,which in the Linde approximation[6] is,

$$R_c = \frac{2\sigma}{\Delta p}\ or\ \sigma = \frac{3\Delta F_c}{4\pi R_c^2} \quad (2)$$

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II. THE STATISTICAL MODEL

In the approximation scheme of the Ramanathan et.al [4, 5], the expression for the density of states, with the modification of Thomas-Fermi model [6], for the quarks and gluons(q,g) in this model is:

$$\rho_{q,g}(k) = (\nu/\pi^2)\{(-V_{conf}(k))^2(-\frac{dV_{conf}(k)}{dk})\}_{q,g}, \quad (3)$$

where k is the relativistic four-momentum of the quarks and the gluons, ν is the volume of the fire ball taken to be a constant in the first approximation and V is a suitable confining potential relevant to the current quarks and gluons in the QGP [4] given by:

$$V_{\text{eff}}(k)k^3 = (1/2k)\gamma_{g,q}\ g^2(k)T^2 - m_0^2/2k\ . \quad (4)$$

where m is the mass of the quark which we take as zero for the up and down quarks and 150 MeV for the strange quarks.The g(k) is the QCD running coupling constant given by

$$g^2(k) = (4/3)(12\pi/27)\{1/\ln(1 + k^2/\Lambda^2)\}\ , \quad (5)$$

where Λ is the QCD scale taken to be 150 MeV.In fact, eq.(3) may be considered as the starting ansatz of the model without further elaboration as far as our calculational program is concerned.

The model has a natural low energy cut off at:

$$k_{min} = V(k_{min})\ or\ k_{min} = (\gamma_{g,q}N^{\frac{1}{3}}T^2\Lambda^2/2)^{1/4}, \quad (6)$$

with $N = [(4/3)(12\pi/27)]^3$.

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The free energy of the respective cases i(quarks,gluons,interface etc.) for Fermions and Bosons(upper sign or lower sign) can be computed using the following expression:

$$F_i = \mp T g_i \int dk \rho_i(k) \ln(1 \pm e^{-(\sqrt{m_i^2+k^2})/T}) \ , \quad (7)$$

With the surface free-energy given by a modified Weyl[7] expression:

$$F_{interface} = \gamma T \int dk \rho_{weyl}(k) \delta(k-T) \ , \quad (8)$$

where the hydrodynamical flow parameter for the surface is:

$$\gamma = \sqrt{2} \times \sqrt{(1/\gamma_g)^2 + (1/\gamma_q)^2}, \quad (9)$$

For the pion which for simplicity represents the hadronic medium in which the fire ball resides ,the free energy is:

$$F_\pi = (3T/2\pi^2)\nu \int_0^\infty k^2 dk \ln(1 - e^{-\sqrt{m_\pi^2+k^2}/T}) \ . \quad (10)$$

For the sake of testing the robustness of our statistical model we have done the calculations using the Cornell potential [9,10] and Richardson Potential [9a] in our formalism.

For Gluon Sector, the cornell potential in momentum space is

$$V(k) = \frac{12\pi\alpha_s}{k^2} + \frac{8\pi C\sigma}{k^4} \quad (11)$$

where α_s=Strong coupling constant =0.09,

C = Casimir Scaling = 9/4, σ = String tension = $8T_c^2$, $T_c = 170\ MeV$ For Quark Sector, the Cornell potential for three flavors in momentum space is

$$V(k) = \frac{16\pi\alpha_s}{3k^2} + \frac{8\pi\sigma}{k^4} \quad (12)$$

where α_s= 0.009, $\sigma = 15T_c^2$, $T_c = 170\ MeV$

For the Richardson potential in the momentum frame we use the form [9a]

$$V(k) = (12\pi/27)(4/3)(1/k^2)(1/\ln(1+k^2/\Lambda^2)) \quad (13)$$

where $\Lambda = 150\ MeV$ and k is four momentum.

We can thus redo our calculations using the above potential forms in our formulation, using the parameter values widely used in the literature.

III. RESULTS AND CONCLUSION

When we compare the numerical results arising from the two different potentials we have considered, the nature of phase transition remains the same in both the potentials' scenarios. The velocity of sound, whose square is the ratio of the entropy and specific heat at constant volume, for the two potentials are not too diferent. While the peshier potential gives almost a contant velocity over a whole range of temperature variation, so does the Cornell potential, except for a distinct kink at around 160 MeV, the expected critical temperature for the transition.However, in the case of the Richardson Potential, the fall in C_s^2 Vs. T is steeper. So, this difference in their prediction could be used to discard one of the potential forms which disagrees with the actual measurement of sound velocity in a QGP fireball formation event in the foreseeble future.The shift in the absolute values of the velocity in the two cases should not hold much physical significance, as the shift seems to be the result of some overall factor in the potentials used. As we have already noted that we could have used any effective potential available in the literature of quark-gluon interaction, our present excercise only strengthens our confidence in the robustness of the statistical model used as two totally unrelated effective potentials, one, the peshier, designed for the deconfined QGP phase and the Cornell for the confined lower momentum regime lead us ti similar scenarios in the QGP-Hadron transition region.Evidently the ideal gas limit of $C_S^2 = \frac{1}{3}$ is obeyed, indicating that the Cornell and Richardson potentials lead to violation of this limit, adding to our already stated reasons for doubting the validity of using these potential