# Temperature fluctuation in quark gluon plasma

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## Introduction

The main aim of the heavy ion collision experiments at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) is to create a new state of matter called quark gluon plasma (QGP). Such a state of matter, *i.e.* QGP might have existed in the early universe after a few micro-second of the big bang. One of the motivation to create and study QGP in the laboratory is to understand the state of the universe in the micro-second old era. The temperature fluctuation in the cosmic microwave background radiation (CMBR) has provided crucial information about the universe when it was about 300,000 years old. A similar approach has been adopted here to study the fluctuations in the energy density and temperature in the system formed in heavy ion collision at relativistic energies (HICRE). The fluctuations in the thermodynamic quantities have been treated as perturbation in the phase space distributions in the hydrodynamic limit. The evolution of these fluctuations has been studied within the ambit of Boltzmann Transport equation (BTE) [1].

## Formalism

The phase space distribution function, $f(\vec{x},\vec{p},t)$ of a system slightly away from equilibrium, at time $t$, position $\vec{x}$, momentum $\vec{p}$ can be written as [1],

$$f(\vec{x},\vec{p},t) = f^{(0)}(p)\{1+\Psi(\vec{x},\vec{p},t)\}, \qquad (1)$$

where $f^{(0)}(p)$ is the equilibrium phase space distribution function and $\Psi(\vec{x},\vec{p},t)$ is the fractional deviation from $f_0(p)$ which can be used to estimate the fluctuations in various thermodynamic quantities in the system. The evolution of $\Psi$ is governed by BTE.

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The energy momentum tensor, $T^{\mu\nu}$ of the system under study can be written as: $T^{\mu\nu} = \overline{T}^{\mu\nu} + \Delta T^{\mu\nu}$, where the equilibrium (ideal) part, $\overline{T}^{\mu\nu}$ is determined by $f^0(p)$ and the dissipative part, $\Delta T^{\mu\nu}$ is determined by $\Psi$. The fluctuations in various thermodynamic quantities can be expressed in terms of deviation of the components of the stress energy tensor from their mean values. The fluctuation in energy density in momentum space can be written as (see [2] for details):

$$\delta\rho(k_i,t) = \int p^2 dp\, d\Omega\, \epsilon\, f^{(0)}(p)\Psi(k_i,p,n_i,t), \qquad (2)$$

We take the zenith direction along $\vec{k}$ and then angular dependence of $\Psi(k_i,p,n_i,t)$ can be expressed in terms of angles between $\hat{k}$ and $\vec{n}$. Depending on the symmetries of the problem under consideration $\Psi$ can be expressed as series of suitable complete set of orthogonal angular basis functions e.g, for axial symmetry in terms of Legendre polynomials and in absence of such symmetry it can be expressed in terms of spherical harmonics. We define

$$\Delta(k_i,t) = \frac{\delta\rho(k_i,t)}{\bar{\rho}} = -\frac{\delta T^0_0(k_i,t)}{\bar{\rho}}, \qquad (3)$$

For an axially symmetric distribution one can expand $\Psi$ in terms of Legendre polynomials:

$$\Psi(\vec{k},\hat{n},p,t) = \sum_{l=0}^{\infty}(-i)^l(2l+1)\Psi_l(\vec{k},p,t)P_l(\hat{k}.\hat{n}), \qquad (4)$$

Solving for $\Psi$ is relaxation time approximation, we get the fluctuation in energy density at time $t$ as:

$$\Delta(\vec{k},t) = \frac{1}{2}e^{-\frac{(t-t_0)}{\tau}}\sum_{s=0}^{\infty}(-i)^{(s)}(2s+1)F_s(\vec{k},t_0) \int_{-1}^{+1} d\mu P_s(\mu)e^{-ik\mu(t-t_0)}. \qquad (5)$$

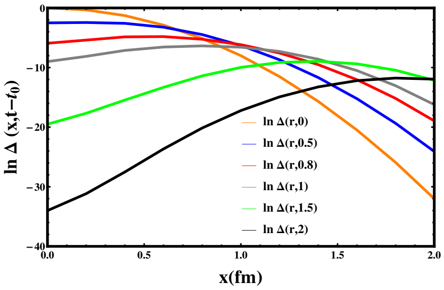


FIG. 1: The evolution of energy density fluctuation in QGP.

## Results

Taking terms upto $s = 2$ in the expression for $\Delta(\vec{k}, t)$ we get,

$$\Delta(\vec{k}, t) = \frac{1}{2} e^{-\frac{(t-t_0)}{\tau}} \sum_{s=0}^{2} (-i)^{(s)} (2s+1) F_s(\vec{k}, t_0) \int_{-1}^{+1} d\mu P_s(\mu) e^{-ik\mu(t-t_0)}. \tag{6}$$

After performing the $\mu$ integration the energy density fluctuation in terms of transport coefficients can be written as (see [2] for details):

$$\begin{aligned}\Delta(\vec{k}, t) =& e^{-(t-t_0)/\tau} [\Delta(\vec{k}, t_0) \frac{\sin k(t-t_0)}{k(t-t_0)} \\ &+ \frac{4}{k} \frac{\chi}{s} \frac{T(\vec{k}, t_0)}{\bar{T}} \{k^2 - ik_l \dot{U}_l(\vec{k}, t_0)\} \\ &\{\frac{\cos k(t-t_0)}{k(t-t_0)} - \frac{\sin k(t-t_0)}{k^2(t-t_0)^2}\} \\ &+ \frac{40}{3} \frac{\eta}{s} \frac{\Theta(\vec{k}, t_0)}{\bar{T}} \{\frac{\sin k(t-t_0)}{k(t-t_0)} \\ &+ \frac{3\cos k(t-t_0)}{k^2(t-t_0)^2} - \frac{3\sin k(t-t_0)}{k^3(t-t_0)^3}\}].\end{aligned} \tag{7}$$

This is the fluctuations in energy density, from which the fluctuations in temperature can be obtained as $\delta\rho/\bar{\rho} = 4\delta T/T$ for $\rho \sim T^4$.

The fluctuation in energy density in position space can be obtained by taking Fourier transformation of Eq. 7 as,

$$\Delta(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \Delta(\vec{k}, t) \exp(i\vec{k}.\vec{x}) \tag{8}$$

If the initial ($t = t_0$) energy density fluctuation, gradient of velocity, viscosity to entropy ($\eta/s$) ratio and temperature of the system in equilibrium are known then Eqs. 7 and 8 can be used to get fluctuations at any time, $t > t_0$.

In Fig. 1 the spatial variation of the fluctuation is plotted at different times. For a case study we have taken $\Delta(k, t_0)$ as a Gaussian and the values of $t_0$ and $\tau$ are taken as: $t_0 = 0.5$ fm/c, $\tau = 1$ fm/c. The results indicate a rapid dissipation and displacement of the peak of the fluctuations with increase in time.

## Summary

In the present work we have studied the time evolution of the fluctuations in thermodynamic quantities using BTE. We establish an explicit relations between the fluctuations and transport coefficients. We have estimated the dissipation of fluctuation in energy density for an initial Gaussian with its peak value equal to unity at the origin. It will be interesting to examine the dissipation of the fluctuations in a more realistic scenario of HICRE [2].

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## References

[1] E. M. Lifshitz and L. P. Pitaevskii, Physical Kinetics (Elsevier, New York, 1987).
[2] G. Sarwar and J. Alam, arXiv:1503.06019 [nucl-th].