Mass-radius relation of magnetized White Dwarfs

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Introduction

Several White Dwarfs (WD) are proposed recently, with masses significantly above the Chandrasekhar limit, known as Super-Chandrasekhar WD, to account for the overluminous Type Ia supernovae [1]. In the present work, Equation of State (EoS) of a completely degenerate relativistic electron gas based on Landau quantization of charged particles in a magnetic field is developed. The mass-radius relations for magnetized WD are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations. The effects of magnetic energy density and pressure contributed by density-dependent magnetic field are treated properly to find the stability configurations of realistic magnetic WD.

EoS for magnetic White Dwarfs

We consider a completely degenerate relativistic electron gas at zero temperature but embedded in a strong magnetic field. We do not consider any form of interactions with the electrons. Electrons, being charged particles, occupy Landau quantized states [2] in a magnetic field. Electrons with spin s and charge $q = -|e|$, the maximum number of particles per Landau level per unit area is $\frac{|e|B(2s+1)}{hc}$ in magnetic field B. On solving Dirac's equation for electrons with spin in an external magnetic field B in z-direction which is uniform and static, energy eigenvalues are given by

$$E_{\nu,p_z} = \left[p_z^2c^2 + m_e^2c^4\left(1+2\nu B_D\right)\right]^{\frac{1}{2}} \qquad (1)$$

where $\nu = n + \frac{1}{2} + s_z$, m_e is electron rest mass and the dimensionless magnetic field defined as $B_D = B/B_c$ is introduced with B_c given by $\hbar\omega_c = \hbar\frac{|e|B_c}{m_ec} = m_ec^2 \Rightarrow B_c = \frac{m_e^2c^3}{|e|\hbar} = 4.414 \times 10^{13}$ gauss. Clearly for the lowest Landau level ($\nu = 0$) the spin degeneracy $g_\nu = 1$ (since only $n = 0$, $s_z = -\frac{1}{2}$ is allowed) and for all other higher Landau levels ($\nu \neq 0$), $g_\nu = 2$ (for $s_z = \pm\frac{1}{2}$). In the present case of magnetic WD, the explicit contributions from the energy density $\varepsilon_B = \frac{B^2}{8\pi}$ and pressure $P_B = \frac{1}{3}\varepsilon_B$ arising due to magnetic field need to be added to the matter energy density and pressure. The expressions for number density, energy density and pressure are as follows:

$$\begin{aligned}
n_e &= \frac{2B_D}{(2\pi)^2\lambda_e^3}\sum_{\nu=0}^{\nu_m} g_\nu x_F(\nu) \\
\varepsilon &= \varepsilon_e + n_e(m_p + fm_n)c^2 + \varepsilon_B \\
&= \frac{2B_D}{(2\pi)^2\lambda_e^3}m_ec^2\sum_{\nu=0}^{\nu_m} g_\nu(1+2\nu B_D)\psi\left(\frac{x_F(\nu)}{(1+2\nu B_D)^{1/2}}\right) \\
&\quad + n_e(m_p + fm_n)c^2 + \frac{B^2}{8\pi} \\
P &= P_e + P_B \\
&= \frac{2B_D}{(2\pi)^2\lambda_e^3}m_ec^2\sum_{\nu=0}^{\nu_m} g_\nu(1+2\nu B_D)\eta\left(\frac{x_F(\nu)}{(1+2\nu B_D)^{1/2}}\right) \\
&\quad + \frac{B^2}{24\pi}, \qquad (2)
\end{aligned}$$

where m_n and m_p are the masses of neutron and proton, respectively and f is the number of neutrons per electron, p_F is the Fermi momentum which is maximum momentum possible at zero temperature, $x_F = \frac{p_F}{m_ec}$ is a dimensionless quantity, $\lambda_e = \frac{\hbar}{m_ec}$ is the Compton wavelength, $\nu_m = \frac{\epsilon_{Fmax}^2 - 1}{2B_D}$ with $\epsilon_{Fmax} = \frac{E_{Fmax}}{m_ec^2}$ the dimensionless maximum Fermi energy and

$$\psi(z) = \int_0^z (1+y^2)^{1/2}dy = \frac{1}{2}[z\sqrt{1+z^2} + \ln(z + \sqrt{1+z^2})],$$

$$\eta(z) = z\sqrt{1+z^2} - \psi(z) = \frac{1}{2}[z\sqrt{1+z^2} - \ln(z + \sqrt{1+z^2})]. \qquad (3)$$

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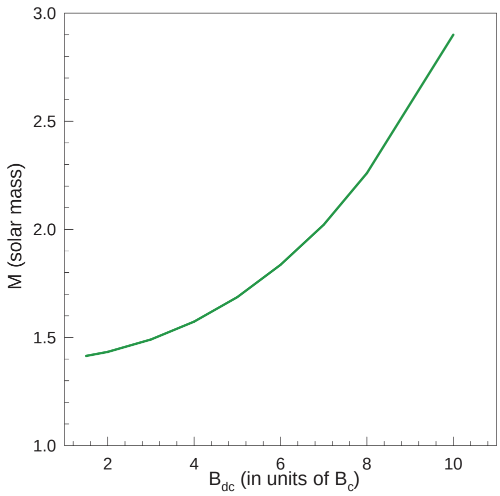

FIG. 1: Plot for masses of magnetized WD as a function of central magnetic field.

Calculations and Results

We perform calculations with varying magnetic field inside WD given by the form [3]

$$B_d = B_s + B_0[1 - \exp\{-\beta(n_e/n_0)^\gamma\}] \quad (4)$$

where B_d (in units of B_c) is the magnetic field at electronic number density n_e, B_s (in units of B_c) is the surface magnetic field and n_0 is taken as n_e(r=0)/10 and β, γ are constants. We choose constants $\beta = 0.8$ and $\gamma = 0.9$, rather arbitrarily but the central and surface magnetic fields once fixed the variations of its profile do not alter the gross results. The maximum central magnetic field strength is kept at $10B_c$ which is 4.414×10^{14} gauss [4] and surface magnetic field at $\sim 10^9$ gauss estimated by observations. In Tables-I,II the results of the calculations are listed for non-magnetic and magnetized WD. In Fig.-1 plot for masses of magnetized WD is shown as a function of central magnetic field.

Summary and Conclusion

The degenerate electron gas at zero temperature under the influence of a density dependent magnetic field is considered where the electrons are Landau quantized and the density of states gets modified due to the presence of the magnetic field which, in turn, modifies the EoS. The magnetic energy density and pressure arising due to the magnetic field are added to those due to degenerate matter. We find that the masses of WD increase with magnitude of the central magnetic field and obtained masses significantly greater than Chandrasekhar limit in the range of $\sim$3 $M_\odot$.

TABLE I: Variations of masses and radii of non-magnetic WD with central number density of electrons which can be expressed in units of 2×10^9 gms/cc by multiplying with 1.6717305×10^6.

n_e (r=0) fm^{-3}	Radius Kms	Mass $M_\odot$
1.0×10^{-5}	917.87	1.3904
5.0×10^{-6}	1126.83	1.3905
4.0×10^{-6}	1202.53	1.3896
2.0×10^{-6}	1466.67	1.3839
1.0×10^{-6}	1779.00	1.3724
8.0×10^{-7}	1890.72	1.3673
4.0×10^{-7}	2275.36	1.3457
1.0×10^{-7}	3233.63	1.2692
1.0×10^{-8}	5482.58	1.0051
1.0×10^{-9}	8721.75	0.5949

TABLE II: Variations of masses and radii of magnetized WD with central number density of electrons. The maximum magnetic field B_{dc} at the centre is listed in units of B_c.

n_e (r=0) fm^{-3}	Radius Kms	Mass $M_\odot$	B_{dc} B_c
4.674017×10^{-6}	1285.91	1.4146	1.5
4.673846×10^{-6}	1344.46	1.4236	1.75
4.674209×10^{-6}	1349.45	1.4339	2.0
4.675374×10^{-6}	1388.04	1.4906	3.0
4.672188×10^{-6}	1438.94	1.5731	4.0
4.670830×10^{-6}	1503.64	1.6863	5.0
4.678118×10^{-6}	1581.27	1.8353	6.0
4.677677×10^{-6}	1663.86	2.0217	7.0
4.665741×10^{-6}	1758.40	2.2601	8.0
4.661657×10^{-6}	1954.44	2.8997	10.

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