Influence of nuclear matter fourth-order symmetry energy on neutron star crust-core phase transition

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Introduction

The equation of state (EoS) of isospin asymmetric nuclear matter (ANM) plays a central role in understanding many critical issues in astrophysics. However, the EoS of ANM, especially, the density dependence of the nuclear symmetry energy $E_s(\rho)$ is poorly known. The Taylor's series expansion of the energy is a popular way of expressing the energy of ANM in the even powers of the asymmetry parameter β. It is an established conclusion derived from many model calculations that the fourth- and higher-order terms in the Taylor's series expansion of energy have negligible contribution. This enables one to express the energy of the ANM in a parabolic expression. The higher order terms though have little contribution to energy of ANM, their role in certain situations have crucial relevance. One of these areas is the prediction of the transition density signifying the onset of the phase transition between the uniform dense liquid core and the solid crust in neutron stars. This is an interesting study made by limited number of workers in the current decade [1-4]. In this work, we shall examine the critical importance of the fourth-order terms in the Taylor's series expansion in the prediction of the crust-core transition density. We shall perform the study in the Non-relativistic mean field approximation using finite range Simple Effective Interaction (SEI) [5] that has been used in the finite nuclei and nuclear matter (NM) studies. We shall use the thermodynamics method to calculate the crust-core phase transition in neutron star.

Formalism

The energy density of ANM, H(ρ,β), can be expressed in Taylor' series expansion as

$$H(\rho,\beta) = H(\rho) + \frac{\beta^2}{2!}\frac{\partial^2 H(\rho,\beta)}{\partial\beta^2}|_{\beta=0} + + \frac{\beta^4}{4!}\frac{\partial^4 H(\rho,\beta)}{\partial\beta^4}|_{\beta=0} + \cdots , \quad \ldots(1)$$

where, H (ρ) is the energy density of SNM at density $\rho(=\rho_n+\rho_p)$, ρ_n and ρ_p being the neutron and proton densities and $\beta = \frac{\rho_n-\rho_p}{\rho_n+\rho_p}$ is the isospin asymmetry parameter.

The core of the neutron star is dominantly composed of β-equilibrated dense n+p+e+μ matter in liquid phase and matter is as a whole charge neutral. The conditions for β-equilibrium on charge neutrality are

$$\mu_n - \mu_p = \mu_e = \mu_\mu \quad \ldots(2)$$

$$Y_p = Y_e + Y_n, \quad \ldots(3)$$

where, μ_i and Y_i, i=n, p, e, μ are the respective chemical potentials and particles fractions. Here n, p, e and μ represent neutron, proton, electron and muon respectively.

The transition density is calculated from the onset of instability of the uniform liquid against small amplitude density fluctuation due to clusterization. This is examined by analyzing the thermodynamical stability conditions in the β-equilibrated dense n+p+e+μ matter which is given by [1,2],

$$V_{th} = \left[2\rho\frac{\partial\varepsilon^b}{\partial\rho} + \rho^2\frac{\partial^2\varepsilon^b}{\partial\rho^2} - \rho^2\left(\frac{\partial^2\varepsilon^b}{\partial\rho\partial Y_p}\right)^2\left(\frac{\partial^2\varepsilon^b}{\partial {Y_p}^2}\right)^{-1}\right] \quad \ldots(4)$$

where, ε^b is the energy per baryon, V_{th} is$-\left(\frac{\partial P}{\partial v}\right)_\mu$ and $v=\frac{V}{B}$ = volume per baryon and P is the baryonic pressure.

Now the stability conditions for the 2nd and 4th order approximation of the energy in Taylor series expansion in Eq.(1) have been examined to calculate the critical transition density by using the finite range SEI

$$v_{eff} = t_0(1 + x_0 P_\sigma)\delta(r) + \frac{t_3}{6}(1 + x_3 P_\sigma)\left(\frac{\rho}{1+b\rho}\right)^\gamma \delta(r) + (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau) f(r), \quad \ldots(5)$$

where, *f(r)* is the functional form of the finite range part and in this work we have taken it to be of Gaussian form. The other terms have their usual meanings. It has in total 11 parameters. For the study of ANM, however, 9 of these 11 interaction parameters are required, namely b, γ, α, ϵ_0^l, ϵ_0^{ul}, ϵ_γ^l, ϵ_γ^{ul}, ϵ_{ex}^l and ϵ_{ex}^{ul}. The connection of the new parameters to the interaction parameters and their determinations has been discussed in Ref.[5].

Results and Discussion

The β-stability conditions in Eq.(2) for the 2nd order and 4th order Taylor series expansion of energy become

$$\mu_n - \mu_p = 4\beta E_{sym,2}(\rho) \quad \ldots(6)$$

$$\mu_n - \mu_p = 4\beta E_{sym,2}(\rho) + 8\beta^3 E_{sym,4} \quad \ldots(7)$$

respectively, where $E_{sym,2}$ and $E_{sym,4}$ are the energy 2nd order 4th order contributions of the Taylor series expansion of the energy density in Eq. (1). Using them in Eq.(2) the proton fraction and lepton fractions at different densities for the 2nd and 4th order approximations are obtained from solutions of Eqs.(2) and (3) simultaneously. Using the value of proton fraction thus obtained we can now calculate the energy per particle of the baryonic part in the β- stable matter. We have performed the calculations for the EoS of SEI that has been recently used [5] in the study of finite nuclei. The EoS of SEI corresponds to γ=1/2 , $E(\rho_0)$ = -16 MeV, ρ_0 =0.157 fm^{-3}, $E_s(\rho_0)$ =35 MeV and $E_s^{'}(\rho_0) = \rho \frac{\partial E_s}{\partial \rho_0}\Big|_{\rho=\rho_0}$ =25.33 MeV.

This value of $E_s^{'}(\rho_0)$ corresponds to the L value 76 MeV. For the EoS the energy per baryon in β stable matter E(ρ,β) is shown in figure-1for the 2nd and 4th order Taylor series approximations as functions of density ρ . It shows that the difference between the two curves is very small that justifies the parabolic approximation of energy in ANM. Although the 4th order contribution has little influence so far as the energy is concerned, its important role in the prediction of the crust core transition densities, ρ_t^{2nd} and ρ_t^{4th}, calculated from Eq.(4) for 2nd and 4th order approximations, respectively, can be seen from Table-1. The comparison of the results of these two approximation shows that the 4th order term lowers the prediction of the transition density by 24% as compared to the 2nd order one. The trend is similar to the one obtained with the relativistic interaction sets [3]. The influence of the L value on the transition density is also examined.

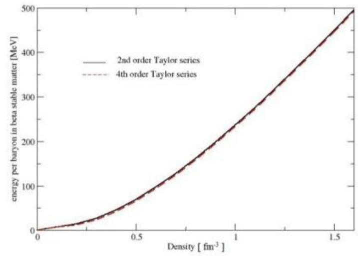

Figure I E(ρ,β) in 2nd and 4th order Taylor series approximations as functions of density ρ

	SEI	FSU Gold	IU-FSU	FSU-I	FSU-II
ρ_t^{2nd}	0.0938	0.089	0.090	0.085	0.088
P_t^{2nd}	1.014	1.316	0.673	0.664	1.010
Y_p^{2nd}	0.0306				
ρ_t^{4th}	0.0739	0.051	0.077	0.069	0.054
P_t^{4th}	0.753	0.321	0.530	0.302	0.236
Y_p^{4th}	0.0493				

Table I Results of the ρ_t^{2nd}(fm^{-3}), ρ_t^{4th}(fm^{-3}), obtained from the thermodynamical method and the corresponding baryonic-pressure P_t^{2nd}(MeV/fm^{-3}), P_t^{4th} (MeV/ fm^{-3}), $Y_p^{2nd}$and Y_p^{4th} for SEI. Results of various RMF interaction sets from Ref.[3] given for comparison.

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